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The statement of the theorem and the main lemma (*)

coNL is defined as {A | the complement A' of A is in NL}.

Theorem 8.27 NL = coNL.

Proof Idea. It is sufficient to prove the following:

PATH∈coNL, i.e. PATH'∈NL (*)

Indeed, assuming that (*) holds, consider any language **A** in NL. Since PATH is NL-complete, there is a log space reduction **f** of **A** to PATH. The same **f**, of course, is also a log space reduction of **A'** to PATH'. By (*), however, PATH'∈NL. Hence **A'**∈NL and thus **A**∈coNL.

For the opposite direction, consider any language **A** in coNL, so that **A'**∈NL. Then there is a log space reduction **f** of **A'** to PATH. Hence **f** is also a reduction of **A** to PATH'. By (*), however, PATH'∈NL. So, **A**∈NL.

To summarize, as long as (*) is true, we have **A∈NL** iff **A∈coNL** for all **A**, meaning that NL=coNL. It remains to understand why (*) holds. This will be done through constructing an NL algorithm **M** for PATH'.