# Kinematics

$$\Delta\vec{r} = \vec{r} - \vec{r}_0 \qquad \vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \qquad \vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$$

$$\text{average speed} = \frac{\text{distance travelled}}{\Delta t}$$

in Cartesian components (3-D):

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$
$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$
$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$$

in plane polar components (2-D):

$$\vec{r} = r\hat{r}$$
$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$$

## Angular Kinematics

$$\Delta\theta = \theta - \theta_0 \qquad \omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} \qquad \alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$$

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

## 1-D motion ($x$ or $\theta$) with constant acceleration ($a$ or $\alpha$)

$$v = v_0 + at \qquad \omega = \omega_0 + \alpha t$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$v^2 - v_0^2 = 2a(x - x_0) \qquad \omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$
$$v_{\text{avg}} = \frac{v + v_0}{2} \qquad \omega_{\text{avg}} = \frac{\omega + \omega_0}{2}$$

## Uniform Circular Motion

$$\vec{v} = R\omega\hat{\theta}$$
$$\vec{a} = -R\omega^2\hat{r} = -\frac{v^2}{R}\hat{r}$$

## Non-uniform Circular Motion

$$\vec{v} = R\omega\hat{\theta}$$
$$\vec{a} = -R\omega^2\hat{r} + R\alpha\hat{\theta}$$

## Relative Velocity

$$\vec{v}_{(\text{C relative to A})} = \vec{v}_{(\text{C relative to B})} + \vec{v}_{(\text{B relative to A})}$$
$$\vec{v}_{(\text{B relative to A})} = -\vec{v}_{(\text{A relative to B})}$$

### Simple Harmonic Motion

The differential equation

$$\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} + {\omega_0}^2 x(t) = 0$$

represents a simple harmonic oscillator, and has solutions of the form

$$x(t) = A\cos(\omega_0 t + \phi)$$

where:

$\omega_0$ is the angular frequency.

$A$ and $\phi$ are arbitrary constants that depend on the initial conditions.

$$f = \frac{1}{T} = \frac{\omega_0}{2\pi}$$ (relation between frequency, period, and angular frequency)

$$\omega_0 = \sqrt{\frac{k}{m}}$$ (angular frequency of a simple mass-spring oscillator)

### Rolling Without Slipping

$$V_{\mathrm{cm}} = \pm R\omega$$

## Dynamics

| | |
|---|---|
| $\sum \vec{F} = m\vec{a}$ | (Newton's 2nd Law for a single particle) |
| $F_G = \frac{Gm_1m_2}{r^2}$ | (gravitational force of attraction between two particles) |
| $F_G = mg$ | (gravitational force on a mass m near the surface of the Earth) |
| $f = \mu_k N$ | (kinetic friction) |
| $f \leq \mu_s N$ | (static friction) |
| $F = -kx$ | (Hooke's Law: linear restoring force) |

### Momentum

| | |
|---|---|
| $\vec{p} = m\vec{v}$ | (momentum of a particle) |
| $\sum \vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$ | (Newton's 2nd Law for a single particle, in terms of momentum) |
| $\vec{F}_{\mathrm{avg}} = \frac{\Delta\vec{p}}{\Delta t}$ | (defintion of average [net] force) |

### Systems of Particles

$$M = \sum_j m_j \qquad \text{(total mass of a system of particles)}$$

$$\vec{P} = \sum_j \vec{p}_j \qquad \text{(momentum of a system of particles)}$$

$$\sum \vec{F}_{\text{ext}} = \frac{\mathrm{d}\vec{P}}{\mathrm{d}t} \qquad \text{(Newton's 2nd Law for a system of particles)}$$

### Center of Mass

$$\vec{R}_{\text{cm}} = \frac{\sum_j m_j \vec{r}_j}{\sum_j m_j} \qquad \text{(center of mass of a system of particles)}$$

$$\vec{P} = M\vec{V}_{\text{cm}} = M\frac{\mathrm{d}\vec{R}_{\text{cm}}}{\mathrm{d}t} \qquad \text{(momentum of a system of particles, in terms of CM)}$$

$$\sum \vec{F}_{\text{ext}} = M\vec{A}_{\text{cm}} = M\frac{\mathrm{d}^2\vec{R}_{\text{cm}}}{\mathrm{d}t^2} \qquad \text{(Newton's 2nd Law for a system of particles, in terms of CM)}$$

### Thrust Equation

$$M\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \sum \vec{F}_{\text{ext}} + \vec{u}_{\text{rel}}\frac{\mathrm{d}M}{\mathrm{d}t}$$

$\vec{u}_{\text{rel}}$ is the velocity of the entering (or leaving) mass particles relative to the main object.

## Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \text{(angular momentum of a particle about the origin)}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \text{(torque about the origin due to a force } \vec{F}\text{)}$$

$$\sum \vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}$$

For a system of particles,

$$\sum \vec{\tau}_{\text{ext}} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}$$

### Rigid Bodies

$$I = \sum_j m_j R_j^2 \qquad \text{(moment of inertia of a rigid body)}$$

$$I_{\parallel} = I_{\text{cm}} + Md^2 \qquad \text{(Parallel Axis Theorem)}$$

$$I_z = I_x + I_y \qquad \text{(Perpendicular Axis Theorem, for a 2-D object in the } xy \text{ plane)}$$