

## Homework 6 – Trigonometric Substitution

For problems 1-7, evaluate the integrals without referring to tables or formulas.

1)  $\int \sqrt{16 - x^2} dx$

2)  $\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1 - x^2}} dx$

3)  $\int \frac{dx}{x\sqrt{x^2 + 16}}$

4)  $\int \frac{dz}{z^3 \sqrt{z^2 - 4}}$

5)  $\int \frac{dx}{\sqrt{x^2 + 4x + 13}}$  Hint: Complete the square first.

6)  $\int \sec^{-1} x dx$

7)  $\int \frac{x^2 dx}{(x^2 + 1)^{\frac{3}{2}}}$

8) Find the average height of a point on the semicircle,  $y = \sqrt{1-x^2}$ , for  $-1 \leq x \leq 1$ .

9) A charged wire creates an electric field at a point P located at a distance D from the wire. The component  $E_{\perp}$  of the field perpendicular to the wire (in N/C) is

$$E_{\perp} = \int_{x_1}^{x_2} \frac{k\lambda D}{(x^2 + D^2)^{\frac{3}{2}}} dx$$

Where  $\lambda$  is the charge density (in C/m),  $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$  and  $x_1$  and  $x_2$  are as in the figure. Suppose that  $\lambda = 6 \times 10^{-4} \text{ C/m}$ , and  $D = 3\text{m}$ . Find  $E_{\perp}$  if  $x_1 = -15\text{m}$  and  $x_2 = 15\text{m}$ .

