Formulae

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10:03 AM

Cobb-Douglas function:

$f(x,y) = Cx^{A}y^{1-A}$ A & C are constants.

Level curves:

ex. $f(x,y) = 60x^{\frac{3}{4}}y^{\frac{1}{4}} = 600$ (level curve)

Partial derivative of $f(x,y) = \frac{\partial d}{\partial x}$

- $f(a+h, b) - f(ab) \approx \frac{\partial f}{\partial x}(a,b)h$

- $f(a, b+k) - f(a,b) \approx \frac{\partial f}{\partial y}(a,b)k$

- second partial derivative

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}(\sim)\right)$$

- $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}(\sim)\right)$

- $\frac{\partial f}{\partial x} x^2 e^{3x} = 2x \cdot e^{3x} + x^2 \times 3 \cdot e^{3x}$

- $\frac{\partial f}{\partial y}(xze^{yz}) = xz \times e^{yz} \times z = xz^2e^{yz}$ (no y in e)

- $\frac{\delta f}{\partial z}(xze^{yz}) = xz^{1-1}e^{yz} + xze^{yz} \times y$ (y is next to e)

- $\frac{\partial f}{\partial y} \ln y = \frac{1}{y}$

- $\dfrac{b\frac{\partial f}{\partial x}a - a\frac{\partial f}{\partial x}b}{b^2} = \frac{\partial f}{\partial x}\frac{2x+3y}{9x+3y}$

- If $f(x,y)$ has a relative max or min at $(x,y) = (a,b)$ then $\frac{\partial f}{\partial x}(a,b) = 0$; $\frac{\partial f}{\partial y}(a,b) = 0$

- $D(x,y) = \frac{\partial^2 f}{\partial x^2} \times \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$

  → If $D(a,b)$ & $\frac{\partial^2 f}{\partial x^2} > 0$ then $f(x,y)$ has relative min

  → If $D(a,b) > 0$ and $\frac{\partial^2 f}{\partial x^2}(a,b) < 0$, $f(x,y)$ has relative max at (a,b)

  → If $D(a,b) < 0$ it has no max or min at (a,b)

  → If $D = 0$, no conclusion.

- If $f(x,y,z)$ has relative max/min at (a,b,c) then

$$\frac{\partial f}{\partial x}(a,b,c) = \frac{\partial f}{\partial y}(a,b,c) = \frac{\partial f}{\partial z}(a,b,c) = 0$$

- To maximize/minimize subject $36 - x^2 - y^2$ to a constraint $x + 7y - 25 = 0$

  $f(x,y) = 36 - x^2 - y^2$, $g(x,y) = x + 7y - 25$ and

  $F(x,y,\lambda) = 36 - x^2 - y^2 + \lambda(x + 7y - 25)$

  max/min when $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} = 0$

  solve for x, y, λ

  max/min at $36 - x^2 - y^2$

- lagrange comes useful when you are using 3 or more variables subject to a constraint

7.5 Least squares method

- To find best fit line for points $(1,4), (2,5), (3,8)$.

① $y = Ax + B$

$E_1^2 = (1x + B - 4)^2$ $\quad E = E_1^2 + E_2^2 + E_3^2$

$E_2^2 = (2x + B - 5)^2$

$E_3^2 = (3x + B - 8)^2$

for $f(A,B)$,

$\frac{\partial f}{\partial A} = 28A + 12B = 76$

$\frac{\partial f}{\partial B} = 12A + 6B = 34$

$\rightarrow \quad y = 2x + \frac{5}{3}$

② For $(x_1, y_1) \dots (x_n, y_n)$

$y = Ax + B$

$$A = \frac{N \times \Sigma xy - \Sigma x \times \Sigma y}{N \times \Sigma x^2 - (\Sigma x)^2}$$

- Σx = sum of x coordinates
- Σy = sum of y coordinates
- $\Sigma xy = x_1y_1 + x_2y_2 + \dots + x_ny_n$
- Σx^2 = sum of squares of x coord

$y = mx + b$

$$A = \frac{N \times \Sigma xy - \Sigma x \times \Sigma y}{N \times \Sigma x^2 - (\Sigma x)^2}$$

- Σy = sum of y coordinates
- $\Sigma xy = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n$
- Σx^2 = sum of square of x-coord
- N = no. of points.

$$B = \frac{\Sigma y - A \Sigma x}{N}$$

7.6 - Double Intergals

ex. $\int_1^2 \left(\int_3^4 (y-x)\, dy \right) dx$ Evaluate

1) inner integral first - its dy so y is the variable

$$\left[\frac{1}{2}y^2 - xy\right]_3^4 = \frac{7}{2} - x$$

2) outer integral 2nd - its dx so x is the variable

$$\left[\frac{7}{2} - x\right]_1^2 = \left[\frac{7}{2}x - \frac{1}{2}x^2\right]_1^2 = 2 \longleftarrow$$

*when given boundary pictures, inner integral is top & bottom
outer integral is right and left.

9.1 Integration by Substitution

1) define variable u
2) Transform integral in terms of u
3) " " with respect to du
4) Integrate
5) replace u

$\int (x^2+1)^3\, 2x\, dx$

$u = x^2+1 \quad du = 2x$

$\int u^3\, du$

$= \frac{1}{4}u^4$

$= \frac{1}{4}(x^2+1)^4 \longleftarrow$

9.2 Integration by parts

$$\int u \cdot dv = u \cdot v - \int (v \cdot du)$$

$\boxed{\text{L P E}}$

1st 2nd 3rd

the 1st, 2nd then 3rd will be
u, the rest of equation is dv

9.3 Evaluation of definite integrals

ex. $\int_0^1 2x(x^2+1)^5\, dx$ let $u = x^2+1$, $du = 2x$

$$= \int_{x=0}^{x=1} u^5\, du$$