
Eigen Decomposition and Singular Value Decomposition

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Introduction

- Eigenvalue decomposition
 - Spectral decomposition theorem
 - Physical interpretation of eigenvalue/eigenvectors
 - Singular Value Decomposition
 - Importance of SVD
 - Matrix inversion
 - Solution to linear system of equations
 - Solution to a homogeneous system of equations
 - SVD application
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What are eigenvalues?

- Given a matrix, \mathbf{A} , \mathbf{x} is the eigenvector and λ is the corresponding eigenvalue if $\mathbf{Ax} = \lambda\mathbf{x}$
 - \mathbf{A} must be square the determinant of $\mathbf{A} - \lambda \mathbf{I}$ must be equal to zero
$$\mathbf{Ax} - \lambda\mathbf{x} = 0 \rightarrow \mathbf{x}(\mathbf{A} - \lambda\mathbf{I}) = 0$$
 - Trivial solution is if $\mathbf{x} = 0$
 - The non trivial solution occurs when $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$
- Are eigenvectors are unique?
 - If \mathbf{x} is an eigenvector, then $\beta\mathbf{x}$ is also an eigenvector and $\beta\lambda$ is an eigenvalue
$$\mathbf{A}(\beta\mathbf{x}) = \beta(\mathbf{Ax}) = \beta(\lambda\mathbf{x}) = \lambda(\beta\mathbf{x})$$