# Circuit Symmetry

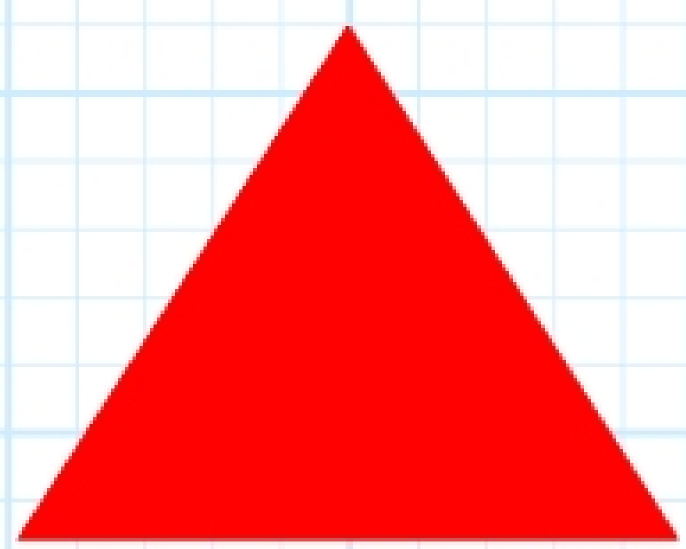

One of the most powerful concepts in for evaluating circuits is that of symmetry. **Normal** humans have a **conceptual** understanding of symmetry, based on an **esthetic** perception of structures and figures.


Évariste Galois

On the other hand, **mathematicians** (as they are wont to do) have defined symmetry in a very precise and unambiguous way. Using a branch of mathematics called **Group Theory**, first developed by the young genius **Évariste Galois** (1811-1832), **symmetry** is defined by a set of operations (a group) that leaves an object **unchanged**.

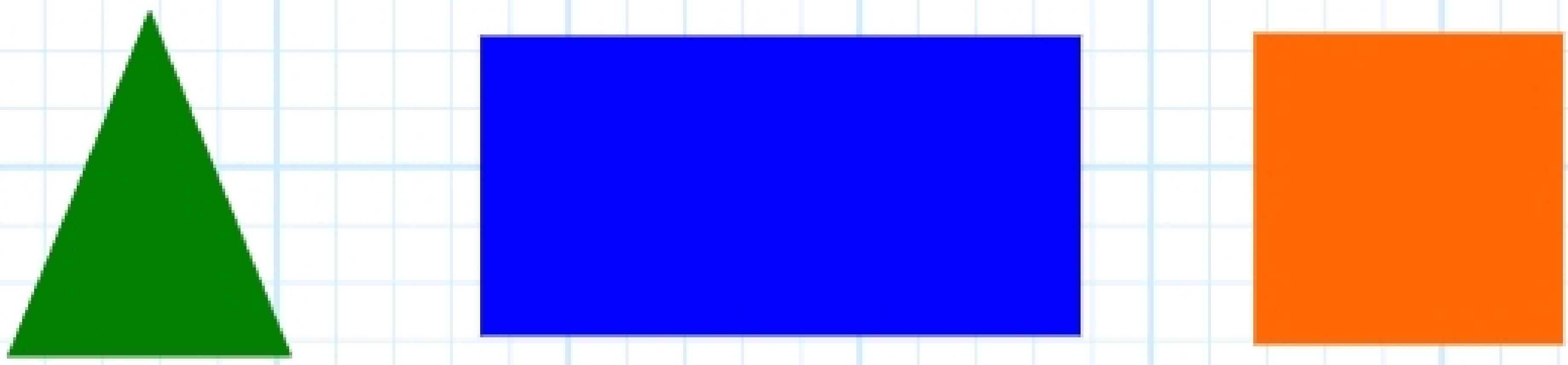
Initially, the symmetric "objects" under consideration by Galois were **polynomial functions**, but group theory can likewise be applied to evaluate the symmetry of **structures**.

For example, consider an ordinary **equilateral triangle**; we find that it is a highly **symmetric** object!

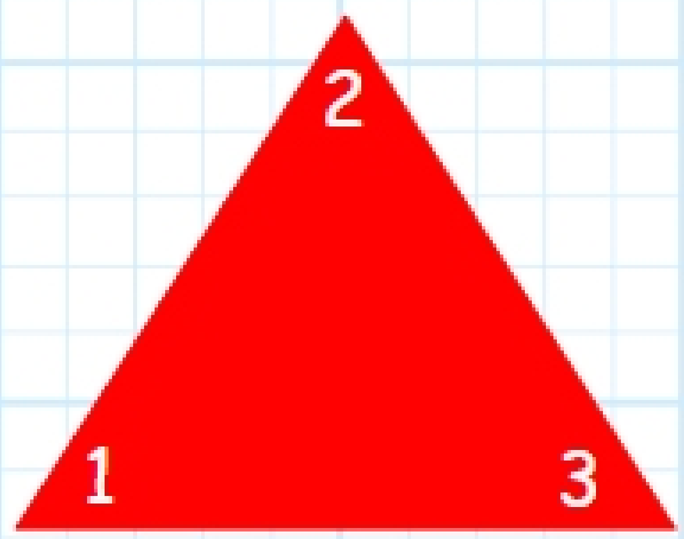


**Q:** ***Obviously*** *this is true. We don't need a mathematician to tell us that!*

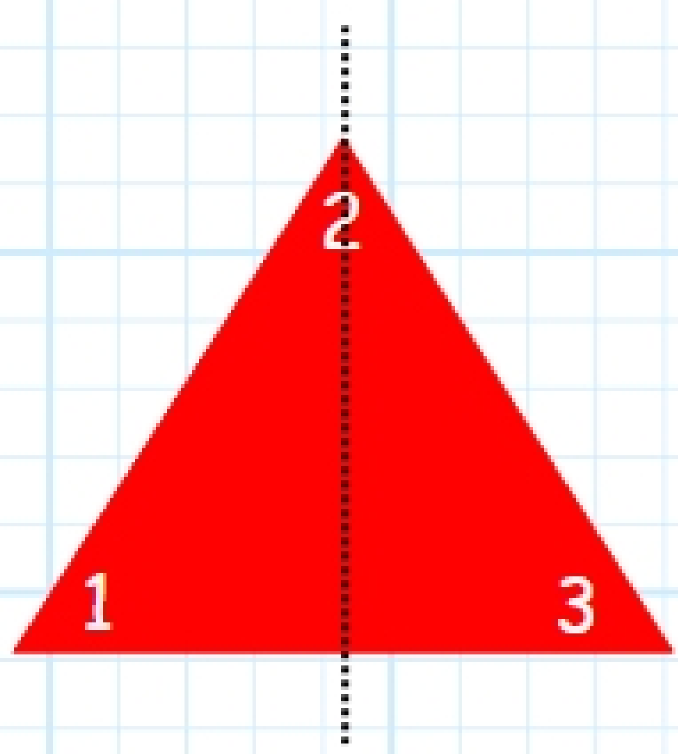
**A:** Yes, but **how** symmetric is it? How does the symmetry of an equilateral triangle **compare** to that of an isosceles triangle, a rectangle, or a square?



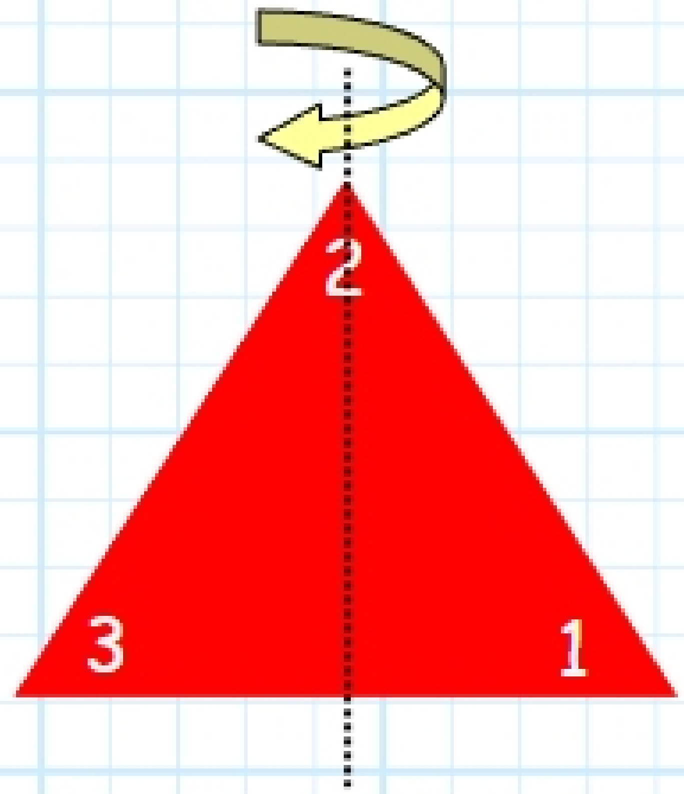
To determine its level of symmetry, let's first label each corner as corner **1**, corner **2**, and corner **3**.




First, we note that the triangle exhibits a plane of **reflection symmetry**:



Thus, if we "**reflect**" the triangle across this plane we get:




Note that although corners 1 and 3 have changed places, the triangle itself remains **unchanged**—that is, it has the same **shape**, same **size**, and same **orientation** after reflecting across the symmetric plane!

Mathematicians say that these two triangles are **congruent.**

Note that we can write this reflection operation as a **permutation** (an exchange of position) of the corners, defined as:

$$1 \rightarrow 3$$
$$2 \rightarrow 2$$
$$3 \rightarrow 1$$

**Q:** *But wait! Isn't there is **more** than just **one** plane of reflection symmetry?*

**A:** Definitely! There are **two more**: