# 7
# Control



The goals of this chapter are to study:

- how to use feedback to control a system;
- how slow sensors destabilize a feedback system; and
- how to model inertia and how it destabilizes a feedback system.

A common engineering design problem is to control a system that integrates. For example, position a rod attached to a motor that turns input (control) voltage into angular velocity. The goal is an angle whereas the control variable, angular velocity, is one derivative different from angle. We first make a discrete-time model of such a system and try to control it without feedback. To solve the problems of the feedforward setup, we then introduce feedback and analyze its effects.

## 7.1 Motor model with feedforward control

We would like to design a controller that tells the motor how to place the arm at a given position. The simplest controller is entirely feedforward in that it does not use information about the actual angle. Then the high-level block diagram of the controller–motor system is


where we have to figure out what the output and input signals represent.

A useful input signal is the desired angle of the arm. This angle may vary with time, as it would for a robot arm being directed toward a teacup (for a robot that enjoys teatime).

The output signal should be the variable that interests us: the position (angle) of the arm. That choice helps later when we analyze feedback controllers, which use the output signal to decide what to tell the motor. With the output signal being the same kind of quantity as the input signal (both are angles), a feedback controller can easily compute the error signal by subtracting the output from the input.

With this setup, the controller–motor system takes the desired angle as its input signal and produces the actual angle of the arm as its output.

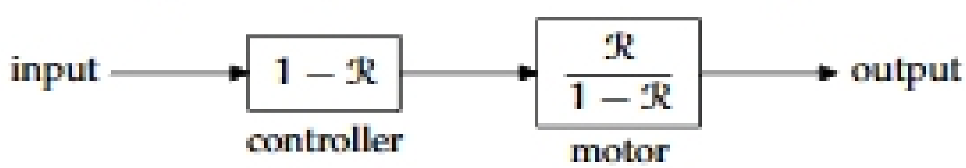
To design the controller, we need to model the motor. The motor turns a voltage into the arm's angular velocity $\omega$. The continuous-time system that turns $\omega$ into angle is $\theta \propto \int \omega \, dt$. Its forward-Euler approximation is the difference equation

$$y[n] = y[n-1] + x[n-1].$$

The corresponding system functional is $\mathcal{R}/(1-\mathcal{R})$, which represents an accumulator with a delay.

> *Exercise 37.* Draw the corresponding block diagram.

The ideal output signal would be a copy of the input signal, and the corresponding system functional would be 1. Since the motor's system functional is $\mathcal{R}/(1-\mathcal{R})$, the controller's should be $(1-\mathcal{R})/\mathcal{R}$. Sadly, time travel is not (yet?) available, so a bare $\mathcal{R}$ in a denominator, which represents a negative delay, is impossible. A realizable controller is $1-\mathcal{R}$, which produces a single delay $\mathcal{R}$ for the combined system functional:



Alas, the $1 - \mathcal{R}$ controller is sensitive to the particulars of the motor and of our model of it. Suppose that the arm starts with a non-zero angle before the motor turns on (for example, the whole system gets rotated without the motor knowing about it). Then the output angle remains incorrect by this initial angle. This situation is dangerous if the arm belongs to a 1500-kg robot where an error of $10^\circ$ means that its arm crashes through a brick wall rather than stopping to pick up the teacup near the wall.

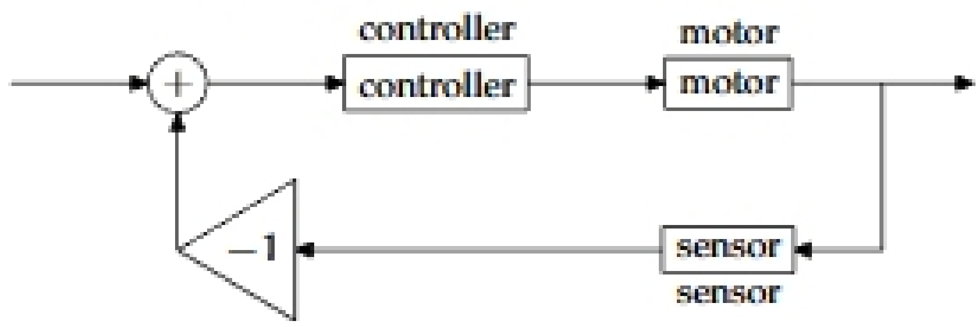
A problem in the same category is an error in the constant of proportionality. Suppose that the motor model underestimates the conversion between voltage and angular velocity, say by a factor of 1.5. Then the system functional of the controller–motor system is $1.5\mathcal{R}$ rather than $\mathcal{R}$. A 500-kg arm might again arrive at the far side of a brick wall.

One remedy for these problems is feedback control, whose analysis is the subject of the next sections.

## 7.2 Simple feedback control

In feedback control, the controller uses the output signal to decide what to tell the motor. Knowing the input and output signals, an infinitely intelligent controller could deduce how the motor works. Such a controller would realize that the arm's angle starts with an offset or that the motor's conversion is incorrect by a factor of 1.5, and it would compensate for those and other problems. That mythical controller is beyond the scope of this course (and maybe of all courses). In this course, we use only linear-systems theory rather than strong AI. But the essential and transferable idea in the mythical controller is feedback.

So, sense the the angle of the arm, compare it to the desired angle, and use the difference (the error signal) to decide the motor's speed:




A real sensor cannot respond instantaneously, so assume the next-best situation, that the sensor includes one unit of delay. Then the sensor's output gets subtracted from the desired angle to get the error signal, which is used