General Physics - E&M (PHY 1308) Lecture Notes

Lecture 011: Handling Tiny Numbers

SteveSekula, 17 September 2010 (created 17 September 2010)

tags: *

Goals of this short lecture

- Heed my warning: be careful when neglecting small numbers

Cautionary Tale: GPS System and Special Relativity

Global Positioning works by using satellites in orbit around the earth to (at least) triangulate your position on the surface of the earth. A hand-held GPS unit (in your phone, car, etc) relies on seeing signals from at least 3 such sateliites. Military-grade GPS can locate a position on the earth to within a few feet. Civilian GPS is good to within a few meters.

QUESTION: Does anybody know what "The Special Theory of Relativity" is, and/or what it tells us?

- ANSWER: it tells us that as you go faster, you observe clocks at rest running more slowly (time runs slowly) and you observe that distances become shorter (space contracts). This has huge effects on anything that relies on precision clocks, like GPS.

But, did you know that Einstein's Special Theory of Relativity predicts a very, very tiny correction to the behavior of these satellites, and that if you neglect the correction then your location on earth will be wrong by 2 more kilometers every day? The satellites travel at about 14,000 km/h; that's 0.001% of the speed of light. Normally, we only worry about special relativity when speeds get to about 5% of that of light, so you might be tempted to ignore special relativity when engineering the GPS system. You'd made a potentially dangerous mistake in doing so.

Why? Well, in the equations you would use to work the GPS problem and determine the size of special relativistic effects, you encounter a formula like this:

$$(a+\delta)^n$$

where a is a number, δ is also a number (whose value is much, much smaller than a, $|\delta| << a$), and n is some power to which the above sum is raised (n can be positive, negative, rational, irrational, etc. - any real number).

SInce δ's value is SO much smaller than a (in the GPS problem, it's value is just 0.001% of a), your instinct might be to toss it. Your calculator's instinct IS to toss it, thus betraying you.

But in the GPS problem, the system is required to be SO precise that you cannot just toss that number out and work the problem without it. It's small value has huge, real consequences.

Your Homework - the Dipole Problem (*SS-11*)

Likewise, in your homework you have a problem involving two dipoles. We know that the dipole force between water molecules in the lungs, though each very tiny, is in fact non-zero and adds up to a big problem for infants without the pulmonary surfactant necessary to reduce the surface tension of water (reduce the dipole force).

Sine the size of the water dipole in *SS-11* is SOOOOOO much tinier than the separation of the two dipoles, you might be tempted to neglect the size of the dipole. You'd make a fatal mistake. Be very careful when working this problem. Since you know that surface tension in the lungs is a real problem, if you get "zero" for the answer to *SS-11* you've obtained a result in direct contradiction with Nature, and Nature always wins.

Let me sketch the problem you'll encounter, and the solution.

You're going to eventually encounter an integral like the following:

$$\int_{x_A+\delta}^{x_B+\delta} f(x)dx = (a+x)^n|_{x_A+\delta}^{x_B+\delta}$$

In both limits of the integral, $|\delta| << x_A, x_B$. You might be tempted to toss the small number and keep the big. But the problem is that the small number matters more than the big number, because at the end of you solution it's the only part of the two that survives, and it's the part that causes infants all that trouble. That small part is the reason small insects can walk on the surface of a pond. That small number matters.

How do I deal with it? *The Binomial Approximation*

Here's how you handle it: the binomial approximation. This approximation let's you simplify the nasty looking polynomial represented by:

$$(a + \delta)^n$$

but ONLY when $|\delta| << a$. The approximation (one version of this is in Wolfson Appendix A) is:

$$(a + \delta)^n \approx a^n \left(1 + n\frac{\delta}{a}\right)$$

Some examples (an in every one, $|y| << 1$):

$$(1 + y)^2 \approx (1 + 2y)$$

$$(1 + y)^{-3} \approx (1 - 3y)$$

$$(1 - y)^4 \approx (1 - 4y)$$

$$(1 - y)^{-2} \approx (1 + 2y)$$