

ECO251 QBA1
THIRD EXAM
Apr 25, 2005

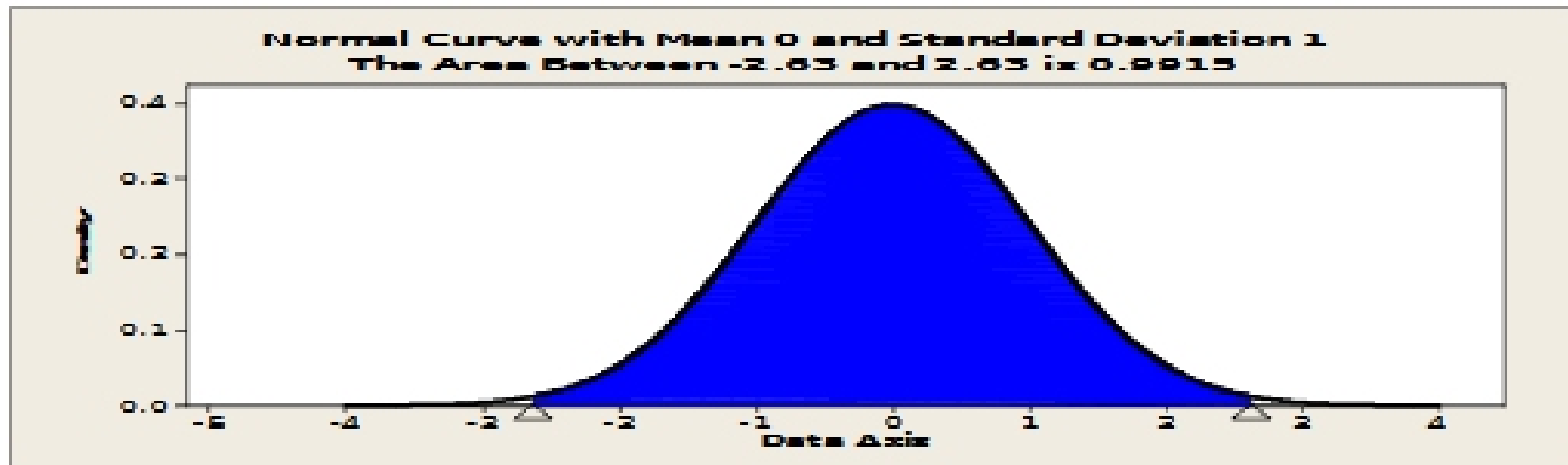
Name: _____
Student Number: _____
Class Time (Circle) 1pm 2pm

Part I: 16 points.

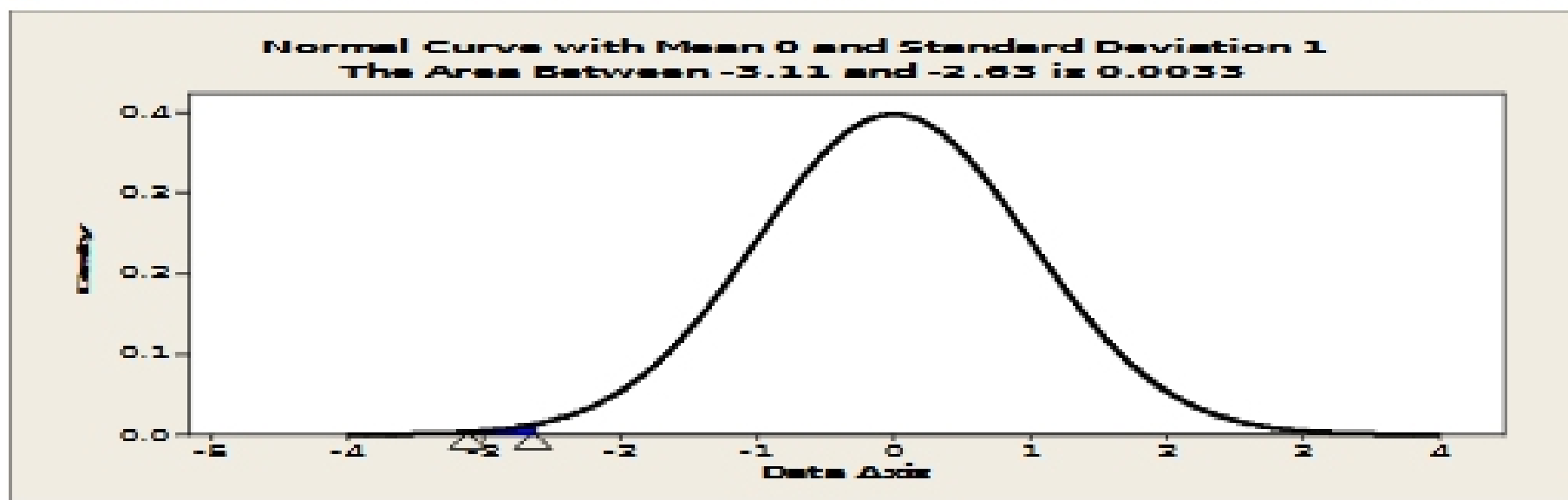
Z follows the standardized Normal distribution ($z \sim N(0,1)$).

Find the following. **Make diagrams! Add a vertical line at zero to these diagrams.**

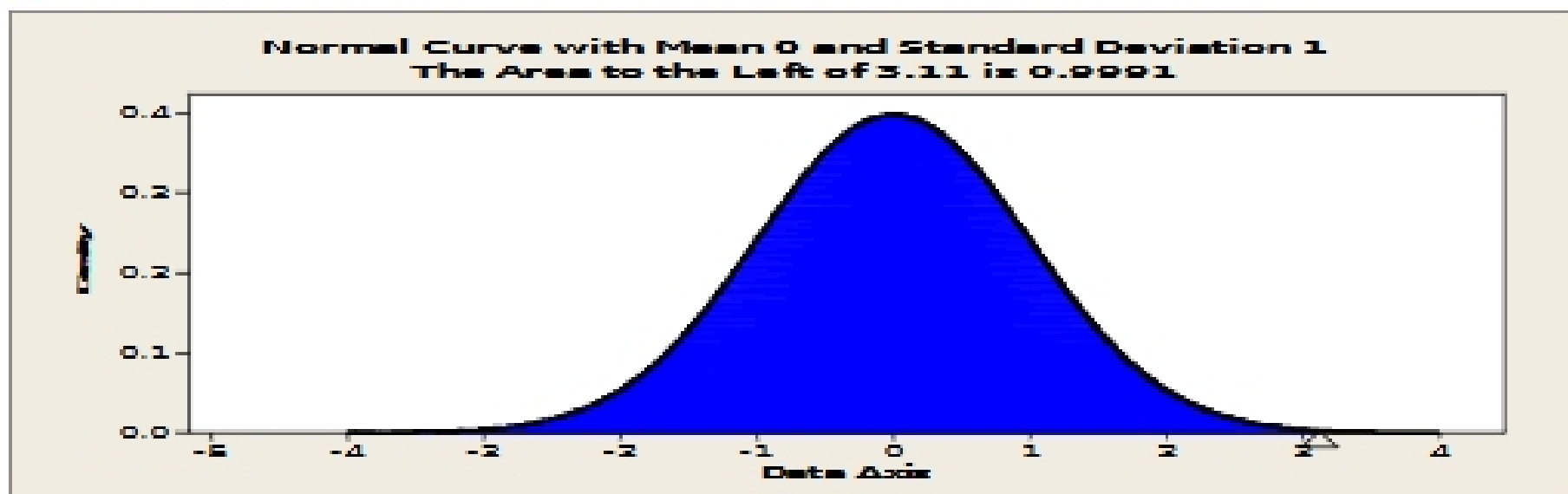
1. $P(-2.63 \leq z \leq 2.63) = P(-2.63 \leq z \leq 0) + P(0 \leq z \leq 2.63) = 2(.4957) = .9914$



2. $P(-3.11 \leq z \leq -2.63) = P(-3.11 \leq z \leq 0) - P(-2.63 \leq z \leq 0) = .4991 - .4957 = .0034$

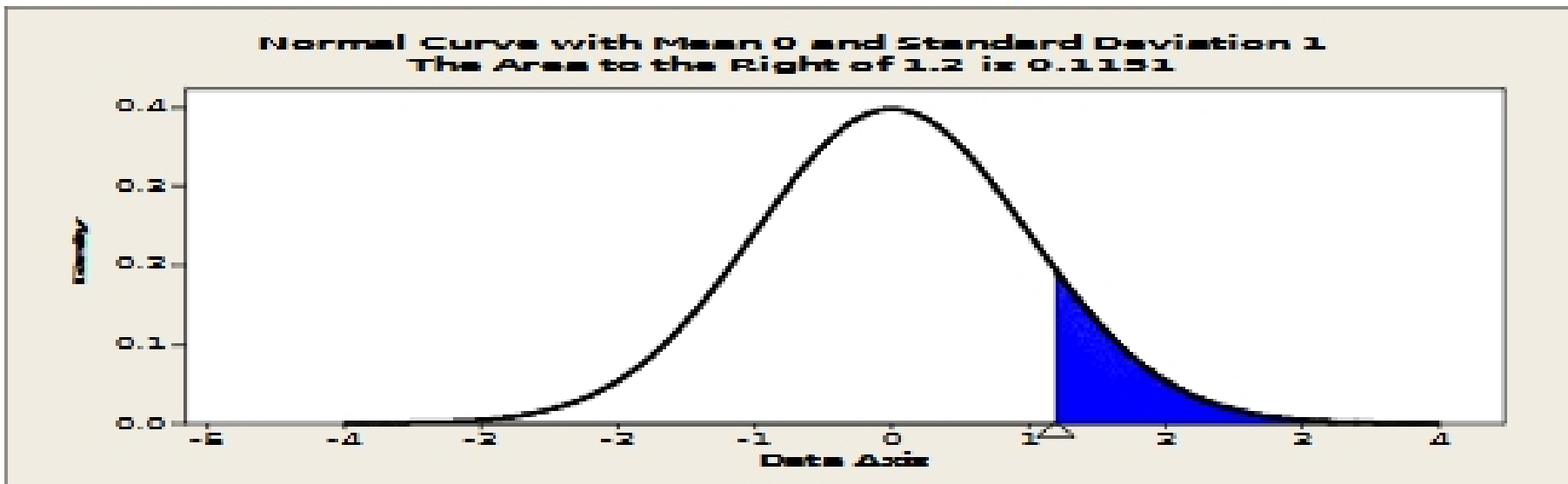


3. $F(3.11) = P(z \leq 3.11) = P(z \leq 0) + P(0 \leq z \leq 3.11) = .5 + .4991 = .9991$



4. $z_{.115}$ **Solution: Make a diagram.** $z_{.115}$ is defined as a point with 11.5% above it and thus $100\% - 11.5\% = 88.5\%$ below it, so it is the 88.5 percentile. The diagram for Z will show an area with a probability of 88.5% below $z_{.115}$. It is split by a vertical line at zero into two areas. The lower one has a

probability of 50% and the upper one a probability of $88.5\% - 50\% = 38.5\%$. The upper tail of the distribution above $z_{.115}$ has a probability of 11.5%, so that the entire area above 0 adds to 50%. From the diagram, we want one point $z_{.115}$ so that $P(z \leq z_{.115}) = .8850$ or $P(0 \leq z \leq z_{.115}) = .3850$. The closest we can come is $P(0 \leq z \leq 1.20) = .3849$. We can say $z_{.115} \approx 1.20$.

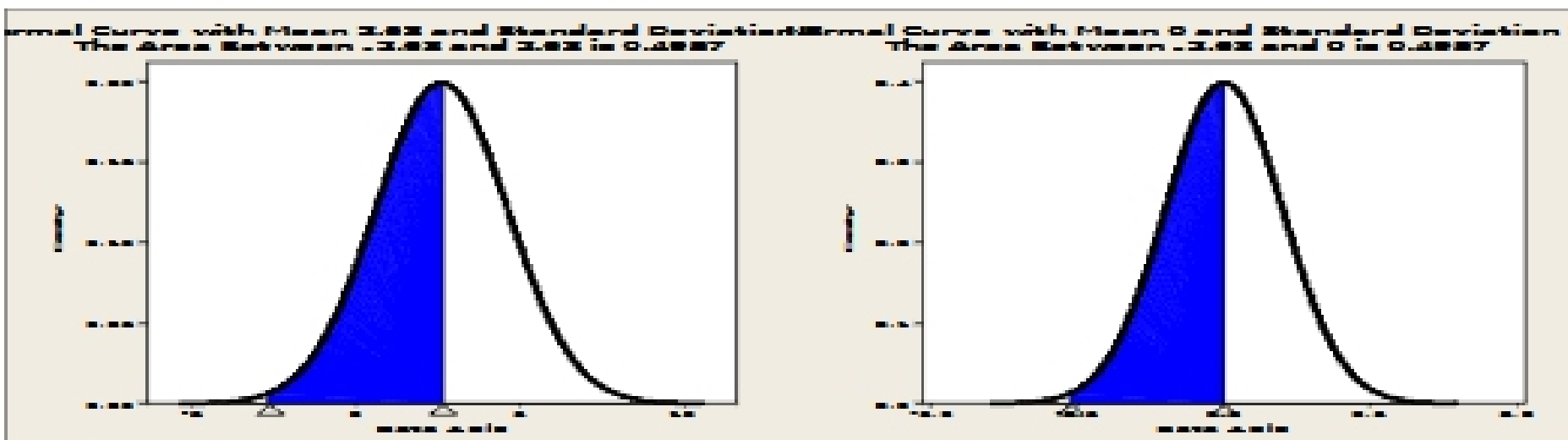


X follows the Normal distribution ($x \sim N(2.63, 2)$). This means $\mu = 2.63$ and $\sigma = 2.00$. Find the following. Make diagrams!

Remember $z = \frac{x - \mu}{\sigma}$. As usual, people made diagrams of X with zero in the middle. Make up your mind! If you are diagramming X , put the mean in the middle; if you are diagramming Z put zero in the middle.

5. $P(-2.63 \leq x \leq 2.63)$

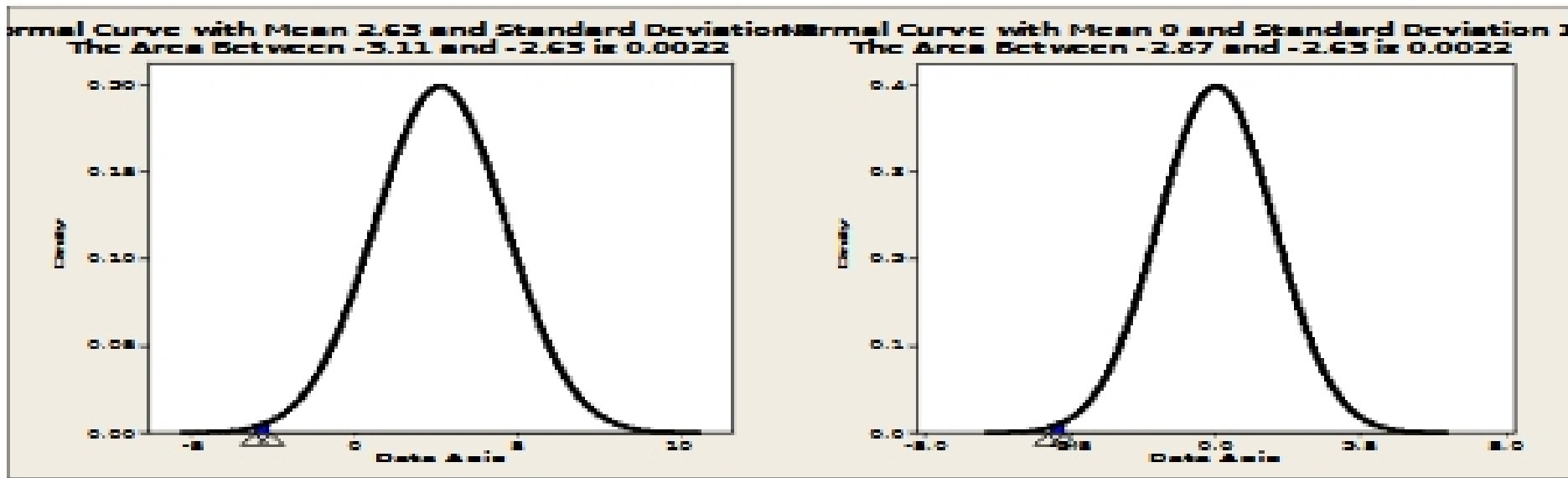
$$= P\left[\frac{-2.63 - 2.63}{2} \leq z \leq \frac{2.63 - 2.63}{2}\right] = P(-2.63 \leq z \leq 0) = .4957$$



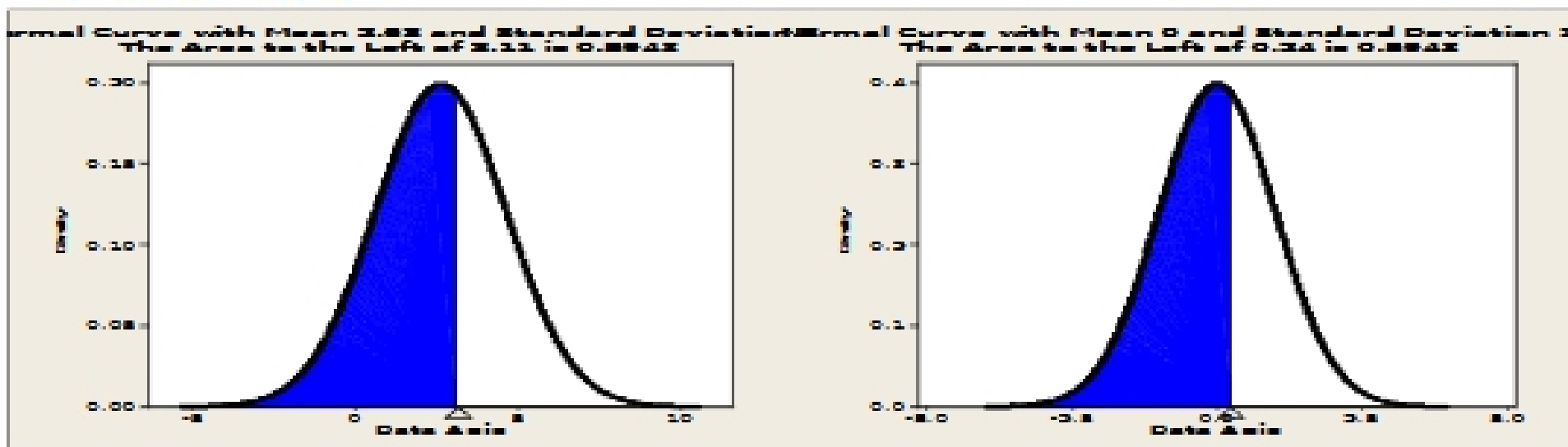
6. $P(-3.11 \leq x \leq -2.63)$

$$= P\left[\frac{-3.11 - 2.63}{2} \leq z \leq \frac{-2.63 - 2.63}{2}\right] = P(-2.87 \leq z \leq -2.63)$$

$$= P(-2.87 \leq z \leq 0) - P(-2.63 \leq z \leq 0) = .4979 - .4957 = .0022$$



$$7. F(3.11) = P(x \leq 3.11) = P\left[z \leq \frac{3.11 - 2.63}{2}\right] = P(z \leq 0.24) = P(z \leq 0) + P(0 \leq z \leq 0.24) \\ = .5 + .0948 = .5948$$



8. $x_{.115}$ In problem 14, we found $z_{.115} \approx 1.20$. The opposite of $z = \frac{x - \mu}{\sigma}$ is $x = \mu + z\sigma$, so

$$x_{.115} = \mu + z_{.115}\sigma = 2.63 + 1.20(2) = 5.03. \text{ Check:}$$

$$P(x \geq 5.03) = P\left[z \geq \frac{5.03 - 2.63}{2}\right] = P(z \geq 1.20) \\ = P(z \geq 0) - P(0 \leq z \leq 1.20) = .5 - .3849 = .1151$$

