Converting from a $N(\mu, \sigma^2)$ Distribution to a $N(0, 1)$ Distribution

If $X \sim N(\mu, \sigma^2)$, then what is Pr ($a < X < b$) for any a, b? Suppose serum total cholesterol is $N(200, 400)$, what is the probability that a randomly selected person from this population will have mild hypercholesterolemia?

That is, what is Pr ($220 < X < 260$)?

We only have tables for $N(0, 1)$ distribution. We must convert the probability statement for $N(\mu, \sigma^2)$ to a probability statement for $N(0, 1)$. Let's create a new random variable Z such that

$$Z = \frac{X - \mu}{\sigma}$$

$Z \sim N(0, 1)$.

Therefore, $\Pr(a < X < b) =$

$$\Pr\left(\frac{a - \mu}{\sigma} < Z < \frac{b - \mu}{\sigma}\right) =$$

$$\Phi\left(\frac{b - \mu}{\sigma}\right) - \Phi\left(\frac{a - \mu}{\sigma}\right)$$

or $\Pr(Z < \frac{b - \mu}{\sigma}) - \Pr(Z < \frac{a - \mu}{\sigma})$

This is the cumulative distribution function for a $N(0, 1)$ as given in column A of Table 3 in the Appendix.

$\mu = 200$, $\sigma^2 = 400$ implies $\sigma = 20$, $a = 220$, $b = 260$.

$$Z_1 = \frac{a - \mu}{\sigma} = \frac{220 - 200}{20} = 1.00$$
$$Z_2 = \frac{b - \mu}{\sigma} = \frac{260 - 200}{20} = 3.00$$

We have converted the probability statement about $N(200, 400)$ to one about $N(0, 1)$. We can evaluate by using the standard normal tables.

$$\begin{aligned}
&\Pr(220 < X < 260) = \\
&\Pr\left(\frac{220 - 200}{20} < Z < \frac{260 - 200}{20}\right) = \\
&\Pr(1.00 < Z < 3.00) = \\
&\Pr(Z < 3.00) - \Pr(Z < 1.00) \quad \text{or} \\
&\Phi(3.00) - \Phi(1.00) = \\
&0.9987 - 0.8413 = 0.1574
\end{aligned}$$

Example: Suppose we have a population of sucrose concentrations that is normally distributed with μ = 65 mg/100 ml and σ = 25 mg/100 ml. What proportion of the population is greater than 85 mg/100 ml?

***X* ~ *N*(65, 625). Find Pr(*X* > 85).**

$$\Pr(X > 85) = \Pr\left(Z > \frac{85 - 65}{25}\right) = \Pr(Z > 0.80)$$

Using Table 3, column B, Pr (*Z* > 0.80) = 0.2119. Therefore, about 21.19% of the population has a sucrose concentration greater than 85 mg/100 ml.

What proportion of the population is less than 45 mg/100 ml?

$$\Pr(X < 45) = \Pr\left(Z < \frac{45 - 65}{25}\right) = \Pr(Z < -0.80)$$

Using Table 3, column A, Pr (Z < -0.80) = 1- Pr (Z < 0.80) = 1 - 0.7881 = 0.2119 or

Using Table 3, column B, Pr (Z < -0.80) = Pr (Z > 0.80) = 0.2119

What proportion of the population is between 45 mg/100 ml and 85 mg/100 ml?