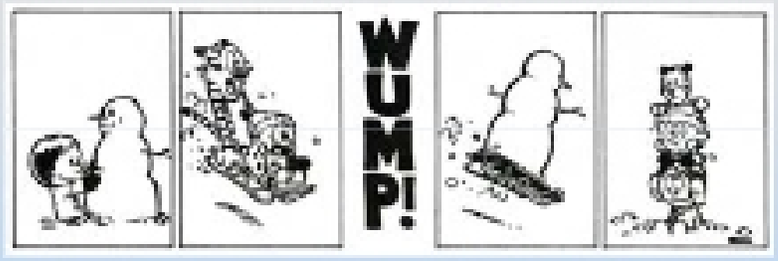
Collision Theory

8.01

Week 09D2

Collisions



Any interaction between (usually two) objects which occurs for short time intervals Δt when forces of interaction dominate over external forces.

- Of classical objects like collisions of motor vehicles.
- Of subatomic particles – collisions allow study force law.
- Sports, medical injuries, projectiles, etc.

Momentum and External Forces

1. Select your system.
2. Identify all forces acting on the system.
3. If there is a non-zero total external force,

$$\vec{\mathbf{F}}_{ext}^{total} = \frac{d\vec{\mathbf{p}}^{sys}}{dt}.$$

4. $\vec{\mathbf{F}}_{ext}^{total} = \vec{\mathbf{0}}$ or collision time is instantaneous implies momentum is constant,

$$\vec{\mathbf{p}}_0^{sys} = \vec{\mathbf{p}}_f^{sys}.$$

Problem Solving Strategies: Momentum

- Identify the objects that compose the system.
- Identify your initial and final states of the system. Draw two momentum diagrams: one for the initial state and the other for the final state.
- Choose symbols to identify each mass and velocity in the system.
- Identify a set of positive directions and unit vectors for each state.
- Decide whether you are using components or magnitudes for your velocity symbols.
- Momentum is a vector!

Momentum Analysis

Since momentum is a vector quantity, identify the initial and final vector components of the total momentum.

Initial State $\vec{\mathbf{p}}_0^{sys} = m_1\vec{\mathbf{v}}_{1,0} + m_2\vec{\mathbf{v}}_{2,0} + \cdots$

x-comp: $p_{x,0}^{sys} = m_1(v_x)_{1,0} + m_2(v_x)_{2,0} + \cdots$

y-comp: $p_{y,0}^{sys} = m_1(v_y)_{1,0} + m_2(v_y)_{2,0} + \cdots$

Final State $\vec{\mathbf{p}}_f^{sys} = m_1\vec{\mathbf{v}}_{1,f} + m_2\vec{\mathbf{v}}_{2,f} + \cdots$

x-comp: $p_{x,f}^{sys} = m_1(v_x)_{1,f} + m_2(v_x)_{2,f} + \cdots$

y-comp: $p_{y,f}^{sys} = m_1(v_y)_{1,f} + m_2(v_y)_{2,f} + \cdots$

Conservation of Momentum

If no external forces are acting on the system, write down the condition that momentum is constant in each direction.

$$p_{x,0}^{sys} = p_{x,f}^{sys}$$

$$m_1(v_x)_{1,0} + m_2(v_x)_{2,0} + \cdots = m_1(v_x)_{1,f} + m_2(v_x)_{2,f} + \cdots$$

$$p_{y,0}^{sys} = p_{y,f}^{sys}$$

$$m_1(v_y)_{1,0} + m_2(v_y)_{2,0} + \cdots = m_1(v_y)_{1,f} + m_2(v_y)_{2,f} + \cdots$$

Planar Collision Theory: Energy

Types of Collisions in Two Dimensions:

Elastic: $K_0^{sys} = K_f^{sys}$

$$\frac{1}{2}m_1v_{1,0}^2 + \frac{1}{2}m_2v_{2,0}^2 + \cdots = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2 + \cdots$$

Inelastic: $K_0^{sys} > K_f^{sys}$

Completely Inelastic: Only one body emerges.

Superelastic: $K_0^{sys} < K_f^{sys}$

Concept Question: Elastic collision

Cart A is at rest. An identical cart B, moving to the right, collides elastically with cart A. After the collision, which of the following is true?

1. Carts A and B are both at rest.
2. Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.
3. Cart A remains at rest and cart B bounces back with speed equal to its original speed.
4. Cart A moves to the right with a speed slightly less than the original speed of cart B and cart B moves to the right with a very small speed.

Concept Question: Inelastic collision

Cart A is at rest. An identical cart B, moving to the right, collides with cart A. They stick together. After the collision, which of the following is true?

1. Carts A and B are both at rest.
2. Carts A and B move to the right with a speed greater than cart B's original speed.
3. Carts A and B move to the right with a speed less than cart B's original speed.
4. Cart B stops and cart A moves to the right with speed equal to the original speed of cart B.

Table Problem: totally inelastic collision

A car of mass m_A moving with speed $v_{A,1}$ collides with another car that has mass m_B and is initially at rest. After the collision the cars stick together and move with speed v_2. What is the change in mechanical energy due to the collision?

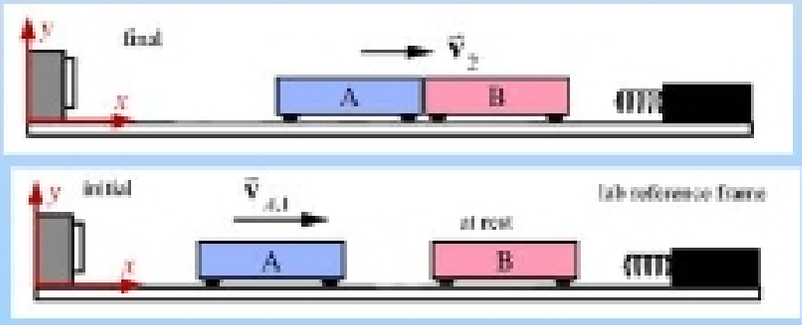


Table Problem: elastic collision

Consider the elastic collision of two carts along a track; the incident cart A has mass m_A and moves with initial speed $v_{A,1}$. The target cart B has mass has mass $m_B = 2\ m_A$ and is initially at rest. Immediately after the collision, the incident cart has final speed $v_{A,2}$ and the target cart has final speed $v_{B,2}$. Calculate the final velocities of the carts as a function of the initial speed .

Example: Elastic Collision in 2-d

In the laboratory reference frame, an "incident" particle with mass m_1, is moving with given initial speed $v_{1,i}$. The second "target" particle is of mass m_2 and at rest. After an elastic collision, the first particle moves off at an angle $\theta_{1,f}$ with respect to the initial direction of motion of the incident particle with final speed $v_{1,f}$. Particle two moves off at an angle $\theta_{2,f}$ with final speed $v_{2,f}$. (i) Find the equations that represent conservation of momentum and energy. Assume no external forces. (ii) Design a strategy to find $v_{1,f}$, $v_{2,f}$, and $\theta_{2,f}$. (You will solve this type of problem on your homework.)