

Giorgi Japaridze

# Theory of Computability

## The Classes L and NL

Section 8.4



## Sublinear complexity

Sublinear (less than linear) time complexity does not make sense (why?). But it does make sense as space complexity. We need to slightly modify our TM model of computation though.

The modification consists in separating the input tape from the work tape. The work tape remains read/write, but the input tape is read-only.

When counting space complexity, we only look at how many cells of the work tape are utilized.

This separation is not artificial. In real life, it is often the case that read-only input is bigger than the computer's memory ("work tape"). CD-ROM is an example.

Or, if we want to focus on fast computations, computer's memory becomes even more limited --- registers only.

Logarithmic space computability can be seen as computability with registers only. Or, more generally, computability with memory that is "much smaller than" input.

## L and NL defined

### Definition 8.17

1. **L** is the class of languages that are decidable in logarithmic space on a deterministic Turing machine. In other words,

$$\mathbf{L = SPACE(\log n)}$$

2. **NL** is the class of languages that are decidable in logarithmic space on a nondeterministic Turing machine. In other words,

$$\mathbf{NL = NSPACE(\log n)}$$