ECO251 QBA1
THIRD EXAM
NOVEMBER 24, 1998

Name ___________________
Section Enrolled: MWF TR 10 11 12:30

Part I. Do all the Following (10+ Points) **Penalty for not doing question 1. Show your work!**

You are following a stock and wish to compare its return to the Dow-Jones index. The return for 9 periods on the stock (y) is compared below to the return on "buying the Dow"(x).

Period	x (Dow)	y (Stock)	y^2	xy
1	12	12	144	144
2	6	15	225	90
3	2	-4	16	-8
4	4	1	1	4
5	4	2	4	8
6	5	-1	1	-5
7	6	-8	64	-48
8	-6	-2	4	12
9	3	3	9	9
Total	36	18	468	206

Note that $\bar{x} = 4$, $s_x = 4.7170$

Compute the following:

1. The sample variance for the stock (4).
2. The sample covariance between the stock and the Dow (2)
3. The sample correlation between the stock and the Dow (2)
4. Interpret the correlation (1)

Answers to questions 5) and 6) must be based on the results in questions 1-4. Do not recompute the answers after changing y !

5. If all the numbers in the y column were higher by 3 (i.e. 15, 18, -1, 4 etc.), what would the variance, covariance and correlation computed above be? (1.5)
6. If all the numbers in the y column were twice as high (i.e. 24, 30, -8, 2 etc.), what would the variance, covariance and correlation computed above be? (1.5)

Solution:

1) $\bar{y} = \frac{\sum y}{n} = \frac{18}{9} = 2$, $s_y^2 = \frac{\sum y^2 - n\bar{y}^2}{n-1} = \frac{468 - 9(2)^2}{8} = 54$ ($s_y = \sqrt{54} = 7.34847$)

2) $\sum xy = 196$, $s_{xy} = \frac{\sum xy - n\bar{x}\bar{y}}{n-1} = \frac{206 - 9(4)(2)}{9-1} = 16.75$.

3) $r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{16.75}{4.7170\sqrt{54}} = 0.4832$

4) The positive sign of r_{xy}, the sample correlation, indicates that x and y tend to move together. If we square r_{xy}, we get approximately .23, which on a zero to one scale indicates a relatively weak relationship.

5) We are leaving x alone, but replacing y by $y+3$. From the syllabus supplement and the outline if $v = cy + d = y + 3$ so that $c = 1$ and $d = 3$

a) $\sigma_v^2 = Var(v) = c^2 Var(y) = c^2\sigma_y^2 = (1)^2\sigma_y^2 = 54$

b) If $w = ax + b$ and $v = cy + d$, $\sigma_{wv} = Cov(ax+b, cy+d) = acCov(x,y)$ and $a = 1$ and $b = 0$ $\sigma_{wv} = acCov(x,y) = (1)(1)(16.75) = 16.75$.

c) $Corr(w,v) = r_{wv} = (sign(ac))r_{xy} = (sign((1)(1))).4471 = (sign(+1)).4832 = .4832$

6) We are leaving x alone, but replacing y by $2y$. From the syllabus supplement and the outline if $v = cy + d = 2y$ so that $c = 2$ and $d = 0$

a) $\sigma_v^2 = Var(v) = c^2 Var(y) = c^2\sigma_y^2 = (2)^2\sigma_y^2 = (4)54 = 216$

b) If $w = ax + b$ and $v = cy + d$, $\sigma_{wv} = Cov(ax+b, cy+d) = acCov(x,y)$ and $a = 1$ and $b = 0$ $\sigma_{wv} = acCov(x,y) = (1)(2)(16.75) = 33.50$.

c) $Corr(w,v) = r_{wv} = (sign(ac))r_{xy} = (sign((1)(2))).4471 = (sign(+2)).4832 = .4832$

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Part II. Do the following problems (do at least 40 points). **Show your work! Note:** You only need 40 points out of the 72 below to get an A+ -. Do the parts that look easiest!

1. The following table represents the joint probability of x and y.

Event	x	0	1	2	$P(y)$	$yP(y)$	$y^2P(y)$
y	2	.25	.10	.05	.40	.80	1.60
	3	.05	.25	.05	.35	1.05	3.15
	4	.15	.10	?	.25	1.00	4.00
$P(x)$		.45	.45	.10	1.00	2.85	8.75

So $E(y) = \mu_y = 2.85$ and $E(y^2) = 8.75$

$xP(x)$ 0+ .45 + .20 = 0.65 $= E(x) = \mu_x$

$x^2P(x)$ 0+ .45 + .40 = 0.85 $= E(x^2)$

a. Fill in the missing number. (1)

b. Are x and y independent? Why?(2)

c. Compute σ_{xy}, the covariance of x and y, and interpret it. (3)

d. Compute ρ_{xy}, the correlation of x and y, and interpret it. (3)

e. Find the probability that $x + y$ is less than 6. (1)

f. (i) Find the distribution of $x + y$. (2)
(ii) Using only the results of a)-d), find the mean and variance of $x + y$.(3)

Solution: a) Since the table must total 1.00, the missing number is 0.

b) x and y are independent if $P(x, y) = P(x) \cdot P(y)$. In the lower right corner $0 \neq (10)(.25)$,
so x and y are not independent .

c) $E(x, y) = \sum\sum xyP(x, y)$

$\sigma_{xy} = Cov(x, y) = E(xy) - \mu_x\mu_y$

= 0(2)(.25) + 1(2)(.10) + 2(2)(.05)
+ 0(3)(.05) + 1(3)(.25) + 2(3)(.05)
+ 0(4)(.15) + 1(4)(.10) + 2(4)(0) = 1.85

$= 1.85 - (0.65)(2.85) = -0.0025$

Negative, so x and y move in opposite directions.

d)

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x\sigma_y} = \frac{(-.0025)}{\sqrt{0.4275}\sqrt{0.6275}} = -\sqrt{\frac{(.0025)^2}{(0.4275)(0.6275)}} = -\sqrt{0.0000232} = -0.0048$$

We measure the strength of a correlation by squaring it. If we square -.0048, we get . 000023. On a zero to one scale, this is tiny, so correlation is very weak.

$Var(x) = \sigma_x^2 = E(x^2) - \mu_x^2 = 0.85 - (0.65)^2 = 0.4275 \quad \sigma_x = \sqrt{0.4275} = 0.6538$

$Var(y) = \sigma_y^2 = E(y^2) - \mu_y^2 = 8.75 - (2.85)^2 = 0.6275 \quad \sigma_y = \sqrt{0.6275} = 0.79215$

e) See below. Since the sums of x and y are all below 6, $P((x + y) < 6) = 1$