CompSci 102 Discrete Mathematics for CS
Spring 2005 Forbes

HW 3

Due Monday, March 28

Please note the following on your assignment: (1) all people with whom you collaborated, (2) resources you used other than the book and class notes, and (3) how much time you spent working on the assignment.

Submit using assignment name *hw03*.

1. (36 pts.) Book Problems

(a) Section 4.3, Exercise 40

(b) Section 5.1, Exercise 38

(c) Section 5.2, Exercise 40

(d) Section 5.3, Exercise 34

2. (10+5 pts.) Monty Hall

Consider a variant of the Monty Hall problem in which there are n doors rather than the usual three. Once again, there is a prize behind one of the doors, and a goat behind each of the other $n-1$. After the contestant chooses a door, the assistant opens one of the unchosen doors to reveal a goat. Under the switching strategy, the contestant then picks one of the $n-2$ remaining doors with equal probability.

(a) What is the probability of winning under the switching strategy (as a function of n)?

(b) **[Extra credit]**

Suppose there are n doors and $m \leq n-2$ prizes (each one behind a different door). Again, the contestant picks a door and the assistant then opens a goat door. (The condition $m \leq n-2$ ensures that this is always possible.) What is the probability of winning under the "sticking" strategy? What is the probability of winning under the "switching" strategy?

3. (20 pts.) A paradox in conditional probability?

Here is some on-time arrival data for two airlines, A and B, into the airports of San Diego and Chicago. (Predictably, both airlines perform better in San Diego, which is subject to less flight congestion and less bad weather.)

	Airline A		Airline B	
	# flights	# on time	#flights	# on time
San Diego	500	453	200	188
Chicago	300	211	900	685

(a) Which of the two airlines has a better chance of arriving on time into San Diego? What about Chicago?

(b) Which of the two airlines has a better chance of arriving on time overall?

(c) Do the results of parts (a) and (b) surprise you? Explain the apparent paradox, and interpret it in terms of conditional probabilities.

4. (20 pts.) Happy families

(a) Consider a collection of families, each of which has exactly two children. Each of the four possible combinations of boys and girls, bg, gb, bb, gg, occurs with the same frequency. A family is chosen uniformly at random, and we are told that it contains at least one boy. What is the (conditional) probability that the other child is a boy? Justify your answer with a precise calculation.

(b) For the same collection of families as in part (a), suppose now that we pick a *child* uniformly at random, and are told that the child is a boy. What is the (conditional) probability that the other child in this family is a boy? Justify your answer with a precise calculation.

(c) Explain the difference in the answers to parts (a) and (b) with reference to the two sample spaces.

5. (14 pts.) Independence

This problem illustrates that events may be pairwise independent but not mutually independent. Two fair dice are thrown. Consider the following three events:

A = the first die is odd;

B = the second die is odd;

C = the sum of the dice is odd.

(a) Show by directly counting sample points that events A, B are independent; that B, C are independent; and that A, C are independent.

(b) Show that the three events are **not** mutually independent.