

## Lab 10: The Atom and the Hydrogen Spectrum

### INTRODUCTION & BACKGROUND:

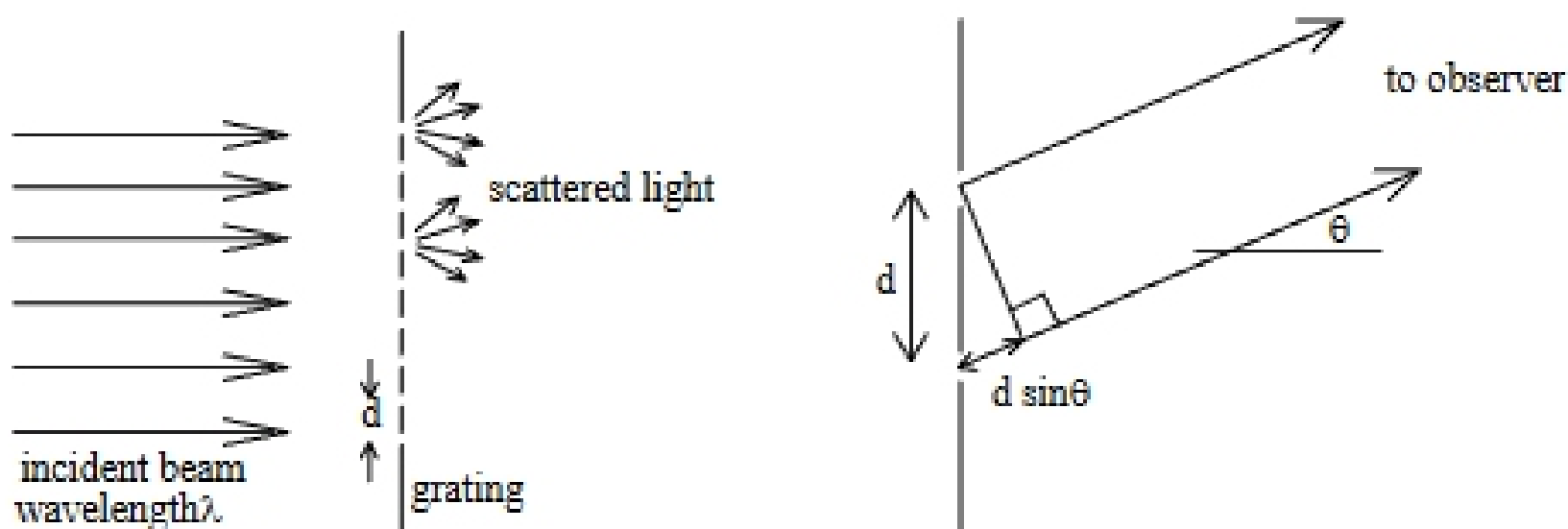
This lab will allow us to look inside an atom. Using tools we have developed over the semester and a simple model of the atom, we will be able to determine which atoms emit which colors of light. Each atom has a unique optical fingerprint and we will use that concept to study the hydrogen atom, and then determine the identity of a mystery atom. This approach is similar to how astronomers determine what stars are made of.

The goals of this lab are to learn how to use a diffraction grating, to understand the Bohr model of the atom and electron orbital structure, energy levels in an atom, and to learn how the orbital structure relates to the spectrum of emitted light.

**Diffraction grating review:** Recall our last lab working with two-slit interference. We observed that the separation of interference maxima on a screen depended on the wavelength  $\lambda$  of light and the distance  $d$  between the slits. Now instead of using just two slits, we will use thousands. The effect is the same as with two slits, except the peaks get much sharper and further apart on the screen (due to smaller slit separation  $d$ ). So we observe sharp bands of light corresponding to specific wavelengths.

A diffraction grating is simply a piece of glass or plastic which has a series of very fine scratches or grooves cut in its surface. The grooves are perfectly straight and parallel and are equally spaced so that there are a fixed number of grooves per millimeter, typically around 500 grooves/mm.

A grating behaves essentially like a multi-slit aperture, that is, a mask with many closely spaced slits. If the number of grooves per length is  $n$  ( $n$  grooves per cm), then the separation between adjacent slits is  $d = 1/n$  (cm per line or simply, cm). Consider what happens when a beam of monochromatic (single-wavelength) light strikes a grating at normal incidence, as shown below. Each groove or slit scatters the light in all forward directions. However, in only certain directions will the light scattered from different grooves interfere constructively, producing a strong beam.



The diagram on the right shows two light rays emerging from adjacent slits in the grating and heading toward an observer (or a point on a screen) at an angle  $\theta$  from the normal (perpendicular) direction. In traveling to the observer, the ray from the lower slit has to travel an extra path distance; this path difference is p.d. =  $d \cdot \sin(\theta)$ . The two rays will interfere constructively only if the path difference is an integer number of wavelengths:

$$(1) \quad d \sin \theta = m\lambda, \quad (m = 0, 1, 2, 3, \dots) \text{ Constructive interference}$$

where  $\lambda$  is the wavelength of the light and  $m$  is any integer. At only these special angles, corresponding to integer  $m$ 's, will the rays from all the slits interfere constructively, producing a bright beam in that direction. In any other direction, the rays from the various slits interfere destructively and produce no light intensity. The integer  $m$  is called the order of the diffraction.

An incident light beam made of a several distinct wavelengths will be split by the grating into its component wavelengths, with each separate wavelength heading in different directions, determined by the condition  $d \sin \theta = m\lambda$ . In this way, the various wavelengths can be determined by measuring the angles.

**Bohr model of the hydrogen atom:** In the 19<sup>th</sup> century, it was known that hydrogen gas, when made to glow in an electrical discharge tube, emitted light at four particular visible wavelengths. In 1885, a Swiss high school teacher named Balmer discovered that the four wavelengths, here labeled  $\lambda_i$  (where  $i = 1, 2, 3, 4$ ) precisely obeyed a curious mathematical relation:

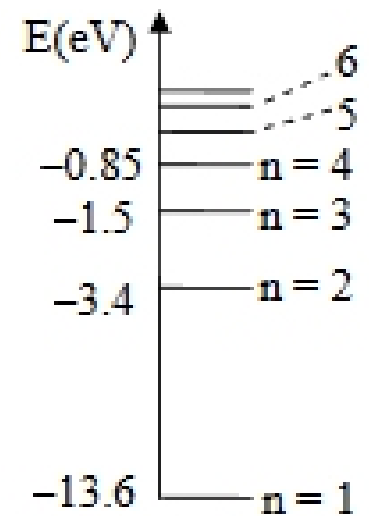
$$(2) \quad \frac{1}{\lambda_i} = R \left( \frac{1}{2^2} - \frac{1}{n_i^2} \right)$$

where  $R$  is a constant, and  $n_i = 3, 4, 5, 6$ . The four wavelengths (or "lines") were henceforth called the Balmer lines of hydrogen. **Why** hydrogen emitted only those visible wavelengths and why the wavelengths obeyed the Balmer formula was a complete mystery.

The mystery was solved in 1913 by the Danish physicist Niels Bohr. According to the Bohr model, the electron orbiting the proton in a hydrogen atom can only exist in certain orbital states labeled with a quantum number  $n$  ( $n=1, 2, 3, 4, \dots$ ). When the electron is in orbit  $n$ , the total energy of the hydrogen atom is given by the formula:

$$(3) \quad E_n = -R \cdot h \cdot c \cdot \frac{1}{n^2} = -\frac{13.6\text{eV}}{n^2},$$

where  $c$  is the speed of light,  $h$  is a constant (Planck's constant), and  $R$  is a number predicted by the Bohr model to be  $R = 1.09737 \times 10^7 \text{ m}^{-1}$ . The different energies  $E_n$  correspond to different orbital states of the electron. Smaller-radius orbits correspond to lower values of  $n$  and lower, more negative, energies. The  $n=1$  state is the lowest possible energy state and is called the ground state.



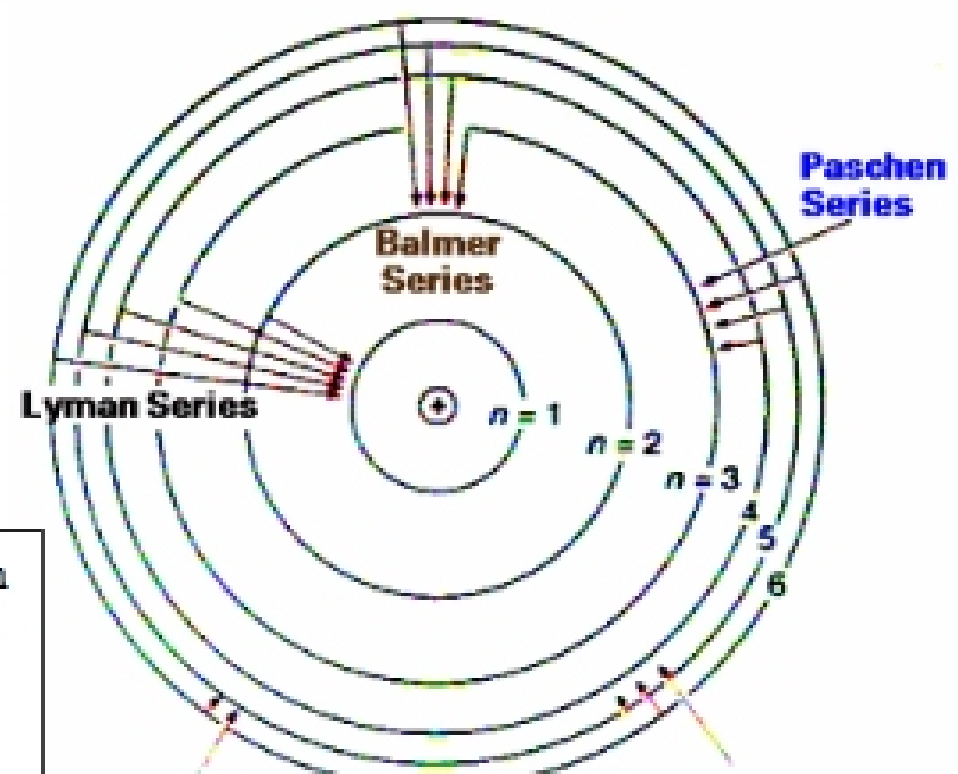
When an electron makes a transition from an initial state of higher energy  $E_i$  to a final state of lower energy  $E_f$ , the atom emits a photon of energy

$$(4) \quad E_\gamma = hf = h \frac{c}{\lambda} = E_i - E_f. \quad \text{Useful number: } hc = 1240 \text{ eV}\cdot\text{nm}$$

Here we have used the expression for the energy of a single photon:  $E = hf$ , where  $h$  is Planck's constant and  $f$  is the frequency of the light. From equations (3) and (4), the wavelength of the emitted photon is related to the initial and final quantum numbers like so:

$$(5) \quad \frac{hc}{\lambda} = E_i - E_f = -R hc \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right), \quad \frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

This is none other than Balmer's formula! Transitions between any pair of states such that  $n_i > n_f$  produces a photon; however, only those transitions with  $n_f = 2$  and  $n_i = 3, 4, 5,$  or  $6$ , happen to produce photons in the visible range of wavelengths.



NOTE: this figure is *not* a picture of the electron orbits of the Bohr model atom; rather it shows the **ENERGY** levels and transitions for various electronic transitions.

Energy level diagram for Hydrogen. Longer lines indicate transitions that emit higher energy photons.