

Paper #3

Due 10/29/02

There are three cities, Alpha, Beta and Gamma sited along one side of a long straight river. Each has been told that they must find a way to treat their wastewater before returning it to the river. Each city has the option of constructing and maintaining its own wastewater treatment plant, or may pump their wastewater along a pipeline to another town that has a treatment plant.

The yearly amortized cost of construction and maintenance of a wastewater treatment plant is $C = 750,000Q^{0.70}$, where Q is the number of cubic feet per second of wastewater that the plant treats. On the other hand, the amortized cost of construction of a pipeline, together with its maintenance costs is $P = 6000Q^{0.50}L$, where Q is the number of cubic feet per second that travel along the pipe, and L is the length of the pipeline, in thousands of feet.

Alpha produces 6 cubic feet of wastewater per second, and is 25,000 feet from Beta. Gamma produces 5 cubic feet of waste per second, and is 40,000 feet from Beta, but in the opposite direction. Beta produces 4 cubic feet of wastewater per second.

Determine how would these towns proceed. In particular, do each of the following.

1. Each town may build its own treatment plant, or may share a plant with a neighbor or neighbors. Determine the costs associated with each possible grouping of towns.
2. Write a game that describes this situation, where the value function represents the amount of money a coalition would save versus the option of all three towns building their own plants.
3. Find the core of this game.
4. Explain why there are exactly six ways to form a coalition, if the order in which the players enter the coalition is significant.
5. For each of the six ways a coalition may be formed, determine the cost to each town after the coalition is formed.
6. On the assumption that the expected division of costs is the average of the six divisions found above, determine the expected distribution of costs.

Rules:

1. The assignment is due at the beginning of class on Tuesday, October 29, 2002.
2. This project is to be completed by teams of 1–4 students, and all students shall make a reasonable contribution to the solution of the problem. Separate from the assignment, each student shall hand in a sheet that describes the work of the group. This will be used in grading; students in the same group may receive different grades.
3. You may not discuss this project with other members of class, or with anyone outside of class. You may (only) discuss the project with the instructor.
4. Aside from the restrictions in (3) above, the use of outside references is acceptable, and indeed encouraged. However, all outside references need to be properly acknowledged.

5. Answers should be a well-written paper that describes the problem and the solution. All of the usual rules of English grammar and composition apply.
6. Papers need to be neat, clean, and paper-clipped or stapled. They do not need to be typed or written in ink, but they must be legible and easily readable.
7. Copying the work of another student or portions of a published work constitutes plagiarism. Plagiarism or any other form of academic dishonesty will be cause for immediate failure of the course.