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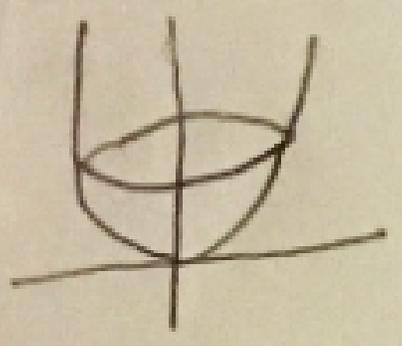
**Math 1172** - Autumn 2015 Name:

**Quiz 3** - Take Home Recitation Instructor:

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

**Problem 1** [4 points] A tank is formed by revolving the graph of $y = 4x^2$ for $0 \le x \le 3$ (in *meters*) about the $y$-axis.

a) If the tank is filled with water to the level (height) of $h$ meters, find the volume of the water in terms of $h$.



$V = A(y)\,dy$

$V = \int_0^h \pi (\sqrt{y}/4)^2\,dy$

$V = \int_0^h y^2/8$

$V = y^2/8 \Big|_0^h$

$V = \pi h^2/8$

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b) If the tank is losing water at the rate of $2\frac{m^3}{s}$, at what rate is the level of the water **falling** when the level is at 1 meter? (Approximate to 2 decimal places.)

$V = \pi h^2/8$

$\frac{dv}{dt} = \pi h/4\ dh/dt$

$\frac{dv}{dt} = 1/4\pi(2\,m^3/s)$

$dv/dt = 0.5\,m^4/s$

c) Given that the density of water is $1000\frac{kg}{m^3}$, find the level of the water when there is $20,000\,kg$ of water in the tank. (Approximate to 2 decimal places.)

$20{,}000\,kg = \int_0^h 1000\,kg/m^3\,(y/4)\pi$

$\frac{20\,m^3}{\pi} = y^2/8 \Big|_0^h$

$h = 7.14\,m$ ✓

d) If the tank contains $20,000\,kg$ of water, how much work is done pumping all of the water to an exit pipe at the top of the tank? (Approximate to 2 decimal places.)

$W = \int_0^{7.14} eg\,A(y)(36-y)$

$= 9800\pi \int_0^{7.14} (y/4)(36-y)$

$9800\pi\left(4y^2 - y^3/12\Big|_0^{7.14}\right)$

$5.34 \times 10^6\,J$

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