Math 128 **Midterm 3** **Spring 2009**

Name:
ID:
Discussion Section:

This exam consists of 16 questions:

- 14 multiple choice questions worth 5 points each
- 2 hand-graded questions worth a total of 30 points.

INSTRUCTIONS: Read each problem carefully and answer the question as written. You may use a non-graphing calculator and a standard sized (no larger than 4×6) index card worth of notes for the exam, but you may use no other aids. Record your answer to the multiple choice questions on the accompanying answer card. Show your work on the written problems and write clearly.

1. Suppose we wish to use the method of Lagrange multipliers to minimize the function $f(x,y)=\frac{1}{2}x^2-3xy+y^2-2$ subject to the constraint $y=x-1$. We can find that this minimum occurs at the point (5, 4) What is the **absolute value** of the Lagrange Multiplier, λ corresponding to this minimum point?

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

(f) 6

(g) 7

(h) 8

(i) 9

(j) 10

Solution: (g). We construct $F(x,y,\lambda)=f(x,y)-\lambda g(x,y)=\frac{1}{2}x^2-3xy+y^2-2-\lambda(y-x+1)$.

Taking partial derivatives, we get: $F_x=x-3y-\lambda$, $F_y=-3x+2y+\lambda$, $F_\lambda=(y-x+1)$.

Setting $F_x=F_y=0$ and solving for λ we get $\lambda=x-3y=-3x+2y$. Plugging our point (5, 4) into these equations, we get $\lambda=-7$.

2. Find the equation of the straight line that minimizes the least-squares error for the points (1,1), (2,3), (3,6).

(a) $y = 2x + \frac{3}{2}$

(b) $y = \frac{5}{2}x - \frac{5}{3}$

(c) $y = \frac{3}{2}x + \frac{3}{2}$

(d) $y = 3x - 1$

(e) $y = \frac{5}{2}x + 1$

(f) $y = 3x - 2$

(g) $y = \frac{3}{2}x - \frac{4}{3}$

(h) $y = \frac{7}{3}x - 3$

(i) $y = 5x - 4$

Solution: (b). We calculate: $N = 3, \sum x = 6, \sum y = 10, \sum x^2 = 14, \sum xy = 25.$

So, $A = \frac{3(25) - (6)(10)}{3(14) - (6)^2} = \frac{15}{6} = \frac{5}{2}$. Then, $B = \frac{10 - (2.5)(6)}{3} = -\frac{5}{3}$.

3. Let $f(x)=\sqrt{x}$. Let $p_2(x)$ be the second Taylor polynomial of f(x) at x = 1. Find $p_2(3)$. (Choose the nearest answer.)

(a) 1.1

(b) 1.2

(c) 1.3

(d) 1.4

(e) 1.5

(f) 1.6

(g) 1.7

(h) 1.8

(i) 1.9

(j) 2.0

Solution: (e). We calculate: $f'(x)=\frac{1}{2x^{1/2}}, f''(x)=-\frac{1}{4x^{3/2}}$.

So, $f(1)=1, f'(1)=\frac{1}{2}, f''(1)=-\frac{1}{4}$. So, $p_2(x)=1+\frac{1}{2}(x-1)-\frac{1/4}{2!}(x-1)^2$.

So, $p_2(3)=1+1-\frac{1}{2}=1.5$.