

- The Three Possibilities
- Summary
  - No solution
  - Infinitely many solutions
  - Unique solution
- Existence of Infinitely Many Solutions
  - More unknowns than equations
  - Missing variable
  - Zero equations
- Example: Three Possibilities with Symbol  $k$

## The Three Possibilities

---

The three possibilities for a linear system, in terms of free variables, rank and nullity, are summarized as follows.

**Table 1. Three possibilities for a linear system.**

No solution	Signal equation $0 = 1$	
Infinitely many solutions	One or more free variables	nullity $\geq 1$
Unique solution	Zero free variables	nullity = 0

## Summary

---

- **No Solution.** There is no solution to a system of equations exactly when a signal equation  $0 = 1$  occurs during the application of swap, multiply and combination rules. We report the system **inconsistent** and announce **no solution**.
- **Infinitely Many Solutions.** The situation of infinitely many solutions occurs when there is **at least one free variable** to which an invented symbol, say  $t_1$ , is assigned. Since this symbol takes the values  $-\infty < t_1 < \infty$ , there are an infinity of solutions. The condition **rank less than n** can replace a reference to the number of free variables.
- **Unique Solution.** There is a unique solution to a system of equations exactly when **zero free variables** are present. This is identical to requiring that the number  $n$  of variables equal the number of lead variables, or **rank = n**.