Linearity and Time Invariance:

If a <u>first-order linear system</u> has no initial stored energy (i.e. initial conditions are all zero $i_L(0+)=v_c(0+)=0$) and several sources are applied, the resultant differential equation will be of the form:

$$\frac{dx}{dt}+\frac{1}{T}x=f_1(t)+f_2(t)+f_3(t)+\cdots f_n(t)$$

The solution is then the sum of the responses to each individual source:

$$x(t)=x_1(t)+x_2(t)+\cdots+x_n(t)$$

If a <u>time invariant system</u> has no initial stored energy and the application of a source signal is delayed by t_0 seconds, then the response will be identical to the non-delayed response except for a delay of t_0 seconds.

Time invariance implies that:

If $\frac{dx}{dt}+\frac{1}{T}x=f(t)$ **results in solution** $x(t)$

then $\frac{dx}{dt}+\frac{1}{T}x=f(t-t_0)$ **results in solution** $x(t-t_0)$.

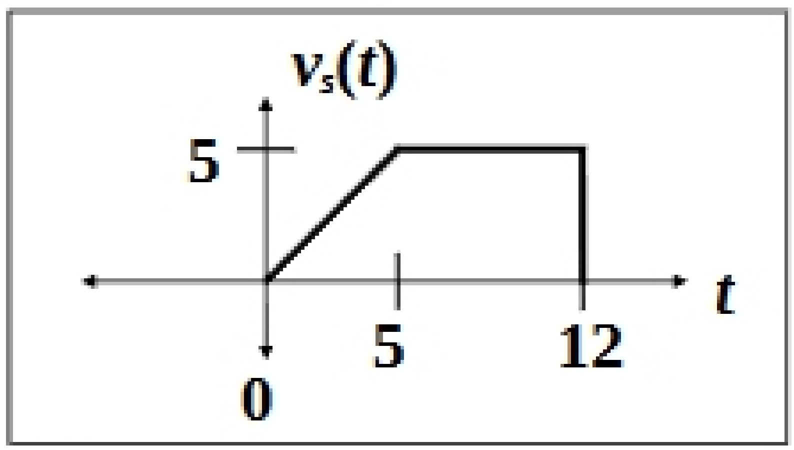
More implications of linearity:

In general for any linear system, if a linear operation is performed on the source (or sources) to create a new source, then the new response is the response of the original source (or sources) with the same linear operation applied. Since differentiation and integration are linear operations, then the derivative of the unit ramp response will be the unit step response, and the derivative to the unit step response will be the unit impulse response.

<u>Examples:</u> For a given linear and time-invariant system with no initial store energy, the unit ramp response is given by:

$$v_{ramp}(t) = \left(6 - 2t - 6\exp\left(-t/2\right)\right)u(t)$$

Find the response if source is a) a unit step, b) a unit impulse, c) function below:



Show a) $v_{step}(t) = \left(-2 + 3\exp\left(-t/2\right)\right)u(t)$

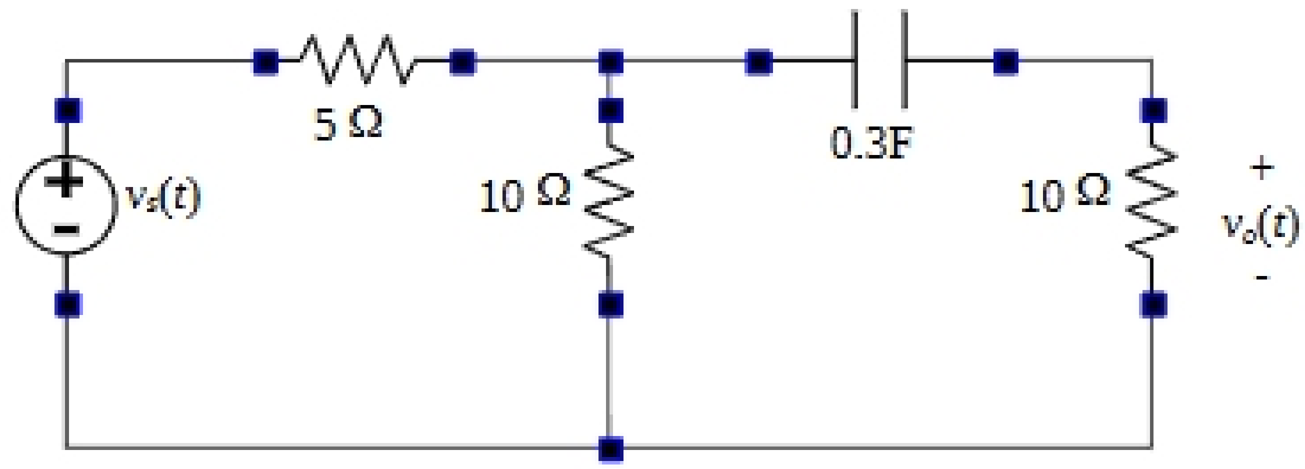
b)

$$v_{imp}(t) = \left(\left(-3/2\right)\exp\left(-t/2\right)\right)u(t) + \delta(t)$$

c) $v_o(t) = \left(6 - 2t - 6\exp\left(-t/2\right)\right)u(t)\cdots$

$$-\left[6 - 2(t-5) - 6\exp\left(-(t-5)/2\right)\right]u(t-5) - 5\left[-2 + 3\exp\left(-(t-12)/2\right)\right]u(t-12)$$

<u>Example:</u> Find v_0 for the circuit below:



a) **when** $v_s(t) = r(t) - 2r(t-1) + r(t-2)$

b) **when** $v_s(t) = 2u(t) - 2u(t-1)$

c) **when** $v_s(t) = 3\delta(t-2)$

Show

a)

$$v_o(t) = 2\left(1 - \exp\left(-t/4\right)\right)u(t) - 4\left[1 - \exp\left(-(t-1)/4\right)\right]u(t-1) + 2\left[1 - \exp\left(-(t-2)/4\right)\right]u(t-2)$$

b) $v_o(t) = \exp\left(-t/4\right)u(t) - \exp\left(-(t-1)/4\right)u(t-1)$