Solid lies inside $x^2+y^2+z^2=1$ $\quad r^2+z^2=4$

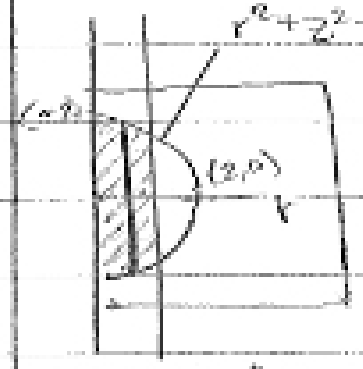
Inside $r=\cos\theta$ $\quad r=\cos\theta$

$r^2=r\cos\theta$

$x^2+y^2=x$

$x^2-x+\frac{1}{4}+y^2=\frac{1}{4}$

Find Volume $\quad (x-\frac{1}{2})^2+y^2=(\frac{1}{2})^2$



$z^2=4-r^2$

$z=\pm\sqrt{4-r^2}$

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{\cos\theta}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

10/31/14

Happy Halloween!

Spherical Coords. $\quad$ Cylindrical

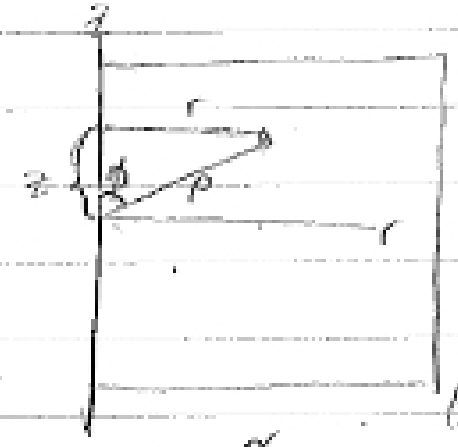
Solid inside sphere $x^2+y^2+z^2=9$ $\quad r^2+z^2=9$

outside cone $x^2+y^2=z^2$ $\quad r^2=z^2$ $\quad r=\pm z$

density $\delta(x,y,z)=e^z$

Compute moment of inertia about z axis.

$$I_z=\iiint_V (x^2+y^2)\,\delta\,dV$$



$\rho\geq 0$

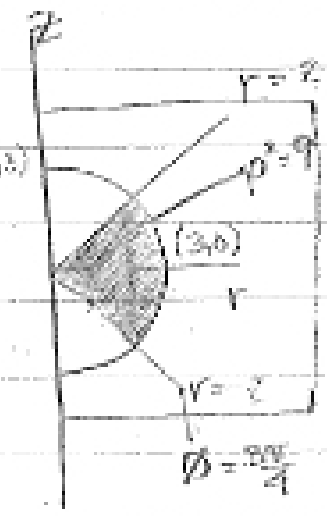
$0\leq\phi\leq\pi$

$0\leq\theta\leq 2\pi$

θ constant

$z=\rho\cos\phi$

$r=\rho\sin\phi$



Next page

$$\int_0^{\sec\phi}\int_{\pi/4}^{2\pi}\int_0^{3}\rho^2\sin^2\phi\, e^{\rho\cos\phi}\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$r=\rho\sin\phi$

$r^2=\rho^2\sin^2\phi$

Jacobian $\rho^2\sin\phi$

$$\int_0^{2\pi}\int_{\pi/4}^{3\pi/4}\int_0^{3}\rho^4\sin^5\phi\, e^{\rho\cos\phi}\,d\rho\,d\phi\,d\theta$$

$$\int\int\int \rho^4(1-\cos^2\phi)(\sin\phi)\,e^{\rho\cos\phi}\,d\rho\,d\phi\,d\theta$$

Cylindrical: r

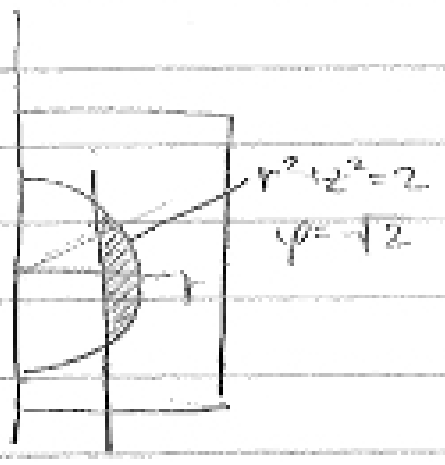
Solid inside sphere $x^2+y^2+z^2=2$

outside cylinder $x^2+y^2=1$

$r^2+z^2=2$

$r^2=1 \quad r=1$

Compute Volume $\iiint_V dV$



$$\int_0^{2\pi}\int_{\csc\phi}^{\sqrt{2}}\int_{\pi/4}^{3\pi/4}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

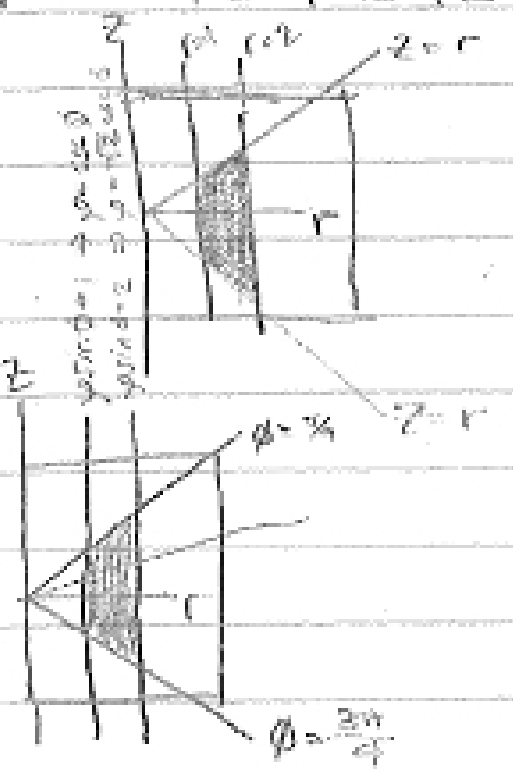
Solid cut from thick walled cylinder $1\le x^2+y^2\le 2$

bdy $z=\pm\sqrt{x^2+y^2}$ $\quad z=\pm r$

$1\le r\le\sqrt{2}$

Cylindrical:

$$\int_0^{2\pi}\int_1^{\sqrt{2}}\int_{-r}^{r} r\,dz\,dr\,d\theta$$



Spherical:

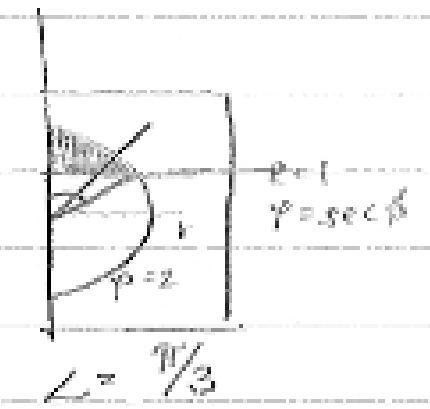
$$\int_0^{2\pi}\int_{\pi/4}^{3\pi/4}\int_1^{\sqrt{2}\csc\phi}\rho^2\sin\phi\,d\rho\,d\phi$$

Solid that is smaller part of $\rho \leq 2$ cut by $z = 1$

Find Volume.

$$\text{Volume} = \iiint_V dV$$

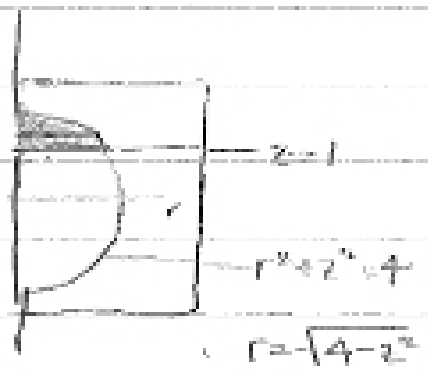
$$\int_0^{2\pi} \int_0^{\pi/3} \int_{\sec\phi}^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$



Cylindrical:

$$\int_0^{2\pi} \int_1^{2} \int_0^{\sqrt{4-r^2}} r \, dr \, dz \, d\theta$$

This one's easier to integrate



Find M_z of part of sphere of radius 2 in first octant.

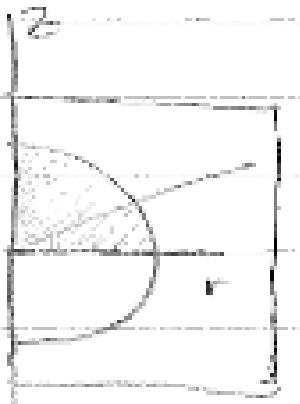
$\delta = 1$

$\rho = 2$

$$M_z = \iiint_V z \, \delta \, dV$$

$$M_z = \int_0^{\pi/2} \int_0^{\pi/2} \int_0^{2} \rho \cos\phi \; \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$M_z = \int_0^{\pi/2} \int_0^{\pi/2} \int_0^{2} \rho^3 \cos\phi \sin\phi \, d\rho \, d\phi \, d\theta$$



* Monday quiz will be cylindrical/spherical