POWER

α--Probability of rejecting H_o when it is true

β--Probability of accepting H_o when it is false—depends on several factors

- ***a.*** **True value of parameter**
- ***b.*** **Hypothesized value or H_o**
- ***c.*** **Value of α**
- ***d.*** **Sample size, *n***

For a fixed α and *n*, and before conducting the test, we can compute many values for β by postulating specific values of parameter if H_o is false.

We want to know for a given hypothesis, how well test controls Type II error. If H_o is false, we want probability of rejecting H_o. This is the complement to β.

1-β -- <u>Power</u> = probability of rejecting H_o when it is false. We must specify a specific alternative value of the parameter to calculate power.

Assume underlying distribution is normal and σ^2 is known.

H_o: $\mu = \mu_0$
H_1: $\mu = \mu_1 < \mu_0$

For a Type I error of α, we reject H_o if $z < z_\alpha$ and accept H_o if $z > z_\alpha$.

Power = Pr(reject H_o | H_o is false) = Pr($z < z_\alpha \mid \mu = \mu_1$)

$$= \Pr\left(\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < z_\alpha \mid \mu = \mu_1\right)$$
$$= \Pr\left(\overline{X} < \mu_0 + z_\alpha * \sigma/\sqrt{n} \mid \mu = \mu_1\right)$$

Under H_1, $\overline{X} \sim N(\mu_1, \sigma^2/n)$

$$\text{Power} = \Phi\left(\frac{\mu_0 + z_\alpha * \sigma/\sqrt{n} - \mu_1}{\sigma/\sqrt{n}}\right)$$
$$= \Phi\left(z_\alpha + \frac{\mu_0 - \mu_1}{\sigma}\sqrt{n}\right) \text{ or } Pr\left(z < z_\alpha + \frac{\mu_0 - \mu_1}{\sigma}\sqrt{n}\right)$$

Reminder: We are testing H_0, the null hypothesis. Under H_0, the rejection region is □.

Under H_1, the area to the left of this point is POWER = 1- β.

Important concept—How likely are we to get a significant difference under the alternative hypothesis? If there is low power, there is little likelihood of getting a significant difference even if H_0 is false.

Example: Suppose we want to study height in a population and we sample 30 individuals. Let μ_0 = 60 inches and σ^2 = 8.44 inches^2. Let α = 0.05.

H_0: $\mu = \mu_0 = 60$
H_1: $\mu = \mu_1 = 58$

What is the power of this test?

$$
\begin{aligned}
\text{Power} &= \Phi\left[z_{\alpha} + \frac{\mu_0 - \mu_1}{\sigma}\sqrt{n}\right] \\
&= \Phi\left[-1.645 + \frac{60-58}{2.9052}\sqrt{30}\right] \\
&= \Phi\left[-1.645 + \frac{2}{2.9052} * 5.4772\right] \\
&= \Phi\left[-1.645 + 3.7706\right] \\
&= \Phi\left[2.1256\right] \text{ or } Pr\left[z < 2.1256\right] \\
&= 0.9834
\end{aligned}
$$

Therefore, the probability of rejecting H_0 at the 5% level of significance if n = 30 is about 98%.

We can calculate power for a test in the opposite direction.

H_0: $\mu = \mu_0$
H_1: $\mu = \mu_1 > \mu_0$

We will reject H_0 if $z > z_{1-\alpha}$

We will accept H_0 if $z \le z_{1-\alpha}$

If $z > z_{1-\alpha}$,

$$\text{then } \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > z_{1-\alpha}$$

$$or\ \overline{X} > \mu_0 + z_{1-\alpha} * \sigma / \sqrt{n}$$

$$\text{Power} = Pr(\overline{X} > \mu_0 + z_{1-\alpha} * \sigma / \sqrt{n} | \mu = \mu_1)$$

$$= 1 - Pr(\overline{X} < \mu_0 + z_{1-\alpha} * \sigma / \sqrt{n} | \mu = \mu_1)$$

$$= 1 - \Phi\left[\frac{\mu_0 + z_{1-\alpha} * \sigma / \sqrt{n} - \mu_1}{\sigma / \sqrt{n}}\right]$$

$$= 1 - \Phi\left[z_{1-\alpha} + \frac{(\mu_0 - \mu_1)}{\sigma}\sqrt{n}\right]$$

$$or\ 1 - Pr\left[z < z_{1-\alpha} + \frac{(\mu_0 - \mu_1)}{\sigma}\sqrt{n}\right]$$

Note: $\Phi(-X) = 1 - \Phi(X)$

and $z_\alpha = -z_{1-\alpha}$

$$\text{Power} = \Phi\left[z_\alpha + \frac{\mu_1 - \mu_0}{\sigma}\sqrt{n}\right] \ or\ Pr\left[z < z_\alpha + \frac{\mu_1 - \mu_0}{\sigma}\sqrt{n}\right]$$

Example: Suppose we are studying birth weight of 100 live births from diabetic mothers. Let $\mu_0 = 120$ ounces, $\sigma = 25$ ounces. Let $\alpha = 0.05$

H_0: $\mu = \mu_0 = 120$
H_1: $\mu = \mu_1 = 125$

What is the power of this test?

$$\text{Power} = \Phi\left[z_\alpha + \frac{\mu_1 - \mu_0}{\sigma}\sqrt{n}\right]$$

$$= \Phi\left[-1.645 + \frac{125 - 120}{25}\sqrt{100}\right]$$

$$= \Phi[-1.645 + 2]$$

$$= \Phi[0.355]\ or\ Pr[z < 0.355]$$

$$= 0.64$$

Therefore, the probability of rejecting H_0 at the 5% level of significance if $n = 100$ is about 64%.

Power depends on 4 factors

1. α