

# Lecture 16: Transformers

Vijay Singh\*

April 10, 2003

## Abstract

Mutual inductance, transformers.

## 1 Motivation

Transmission line voltage is 500 kV.

Voltage of home appliances is 208 V or 120 V.

Need a step-down transformer.

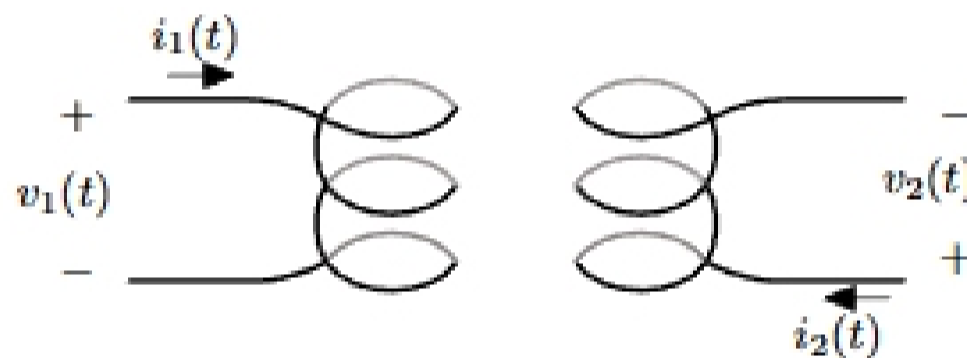


Figure 1: Transformer.

## 2 Equations

By Faraday's law,

$$v_1(t) = N_1 \frac{d\Phi_1}{dt} \quad (1)$$

where,

$$\begin{aligned} N_1 &= \text{Number of turns of the coil,} \\ \Phi_1 &= \text{Flux in Coil 1} \\ &= \Phi_{11} + \Phi_{12} \end{aligned} \quad (2)$$

$\Phi_{11}$  = Flux in Coil 1 due to current in Coil 1

$\Phi_{12}$  = Flux in Coil 1 due to current in Coil 2

$$\text{So, } v_1(t) = N_1 \frac{d\Phi_{11}}{dt} + N_1 \frac{d\Phi_{12}}{dt} \quad (3)$$

---

\*Professor and Chairman, Department of Electrical & Computer Engineering, University of Kentucky, Lexington, KY, USA. E-mail: vsingh@engr.uky.edu. Document prepared using L<sup>A</sup>T<sub>E</sub>X and figures created using Metagraf by Ramprasad Potluri (potluri@engr.uky.edu).

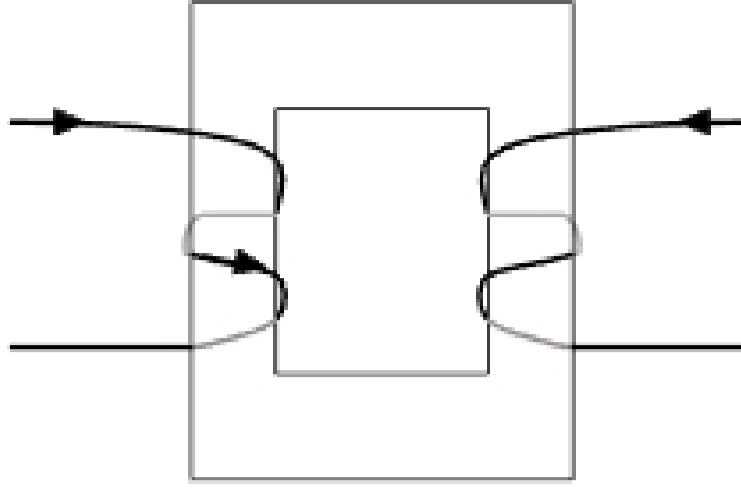


Figure 2: .

But, we know that

$$\Phi_{11} = N_1 i_1 P_{11} \quad (4)$$

$$\Phi_{12} = N_1 i_1 P_{12} \quad (5)$$

where permiances  $P_{11}$  and  $P_{12}$  depend upon the magnetic paths taken by the flux components.

Therefore,

$$v_1(t) = N_1^2 P_{11} \frac{di_1}{dt} + N_1 N_2 P_{12} \frac{di_2}{dt} \quad (6)$$

$$\text{Constant } N_1^2 P_{11} = L_{11} = \text{Self Inductance} \quad (7)$$

$$\text{Constant } N_1 N_2 P_{12} = L_{12} = \text{Mutual Inductance} \quad (8)$$

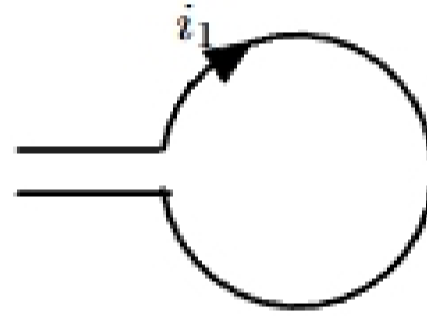


Figure 3: .

Thus,

$$v_1(t) = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt} \quad (9)$$

Similarly,

$$v_2(t) = N_2^2 P_{22} \frac{di_2}{dt} + N_1 N_2 P_{21} \frac{di_1}{dt}$$

$$\text{or, } v_2(t) = L_{22} \frac{di_2}{dt} + L_{21} \frac{di_1}{dt} \quad (10)$$

If the medium is linear, then

$$P_{12} = P_{21} = M$$

$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad (11)$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \quad (12)$$

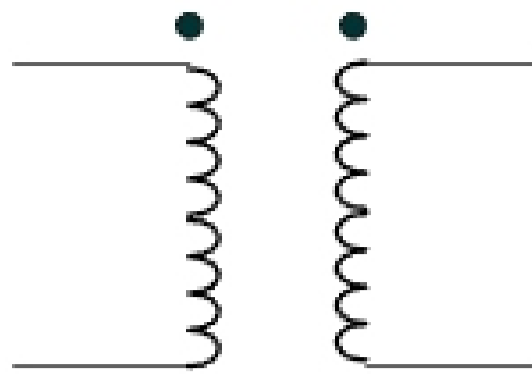


Figure 4: .

If currents enter both dotted terminals of Figure 4, then fluxes produced by both currents will **add**.

Current entering the dot of coil A marks the dotted end of coil B “positive” by virtue of mutual inductance (Figure 5).

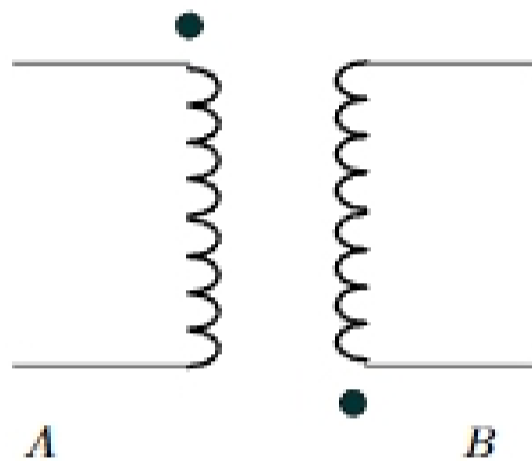


Figure 5: .

### 3 Example

Please see Figure 6.

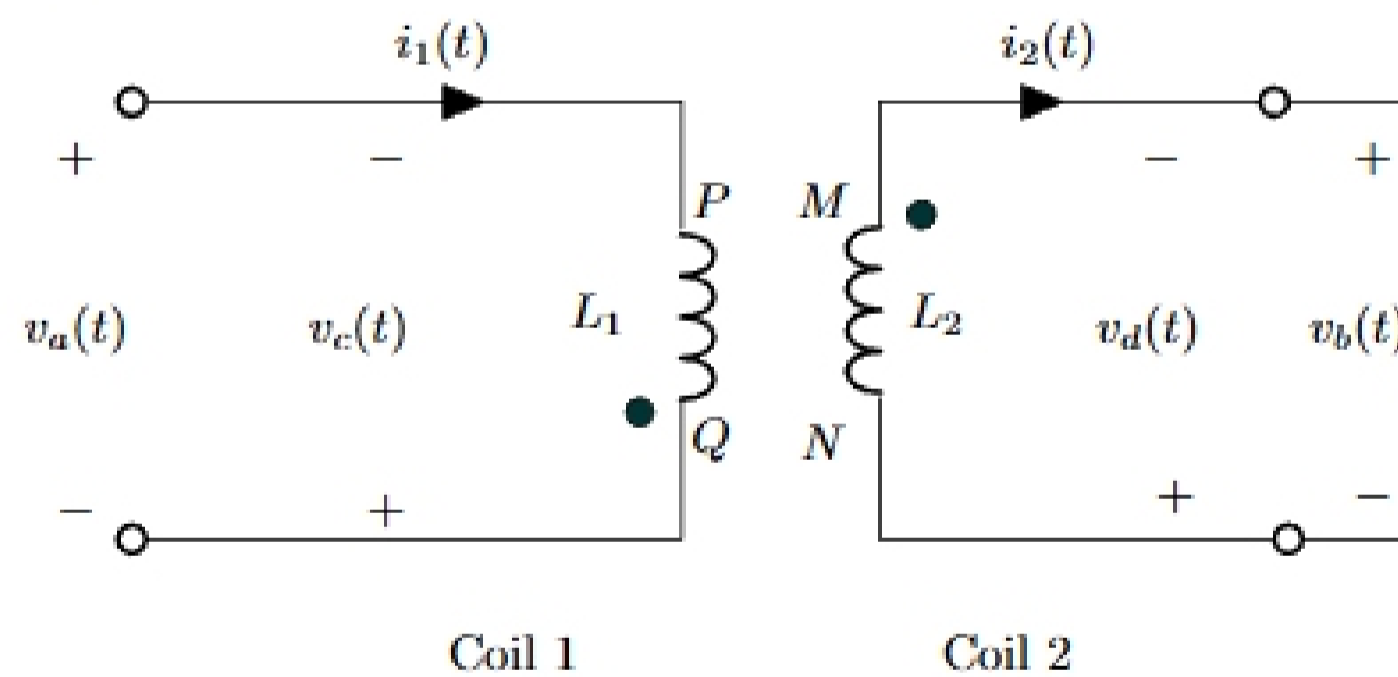


Figure 6: .

In coil 1:

$$v_a(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Here,  $v_a(t) = V_{PQ}$ , and the “+” sign is because current leaving the dot in coil 2 marks the dotted end (Q) of coil 1 **negative**.