

Chapter 7 Dimensional Analysis and Modeling

The Need for Dimensional Analysis

Dimensional analysis is a process of formulating fluid mechanics problems in terms of nondimensional variables and parameters.

1. Reduction in Variables:

If $F(A_1, A_2, \dots, A_n) = 0$,

Then $f(\Pi_1, \Pi_2, \dots, \Pi_{r < n}) = 0$

Thereby reduces number of experiments and/or simulations required to determine f vs. F

F = functional form

A_i = dimensional variables

Π_j = nondimensional parameters

= $\Pi_j(A_i)$

i.e., Π_j consists of nondimensional groupings of A_i 's

2. Helps in understanding physics

3. Useful in data analysis and modeling

4. Fundamental to concept of similarity and model testing

Enables scaling for different physical dimensions and fluid properties

Dimensions and Equations

Basic dimensions: F, L, and t or M, L, and t
F and M related by $F = Ma = MLT^{-2}$

Buckingham Π Theorem

In a physical problem including n dimensional variables in which there are m dimensions, the variables can be arranged into $r = n - \hat{m}$ independent nondimensional parameters Π_r (where usually $\hat{m} = m$).

$$F(A_1, A_2, \dots, A_n) = 0$$

$$f(\Pi_1, \Pi_2, \dots, \Pi_r) = 0$$

A_i 's = dimensional variables required to formulate problem
($i = 1, n$)

Π_j 's = nondimensional parameters consisting of groupings
of A_i 's ($j = 1, r$)

F, f represents functional relationships between A_n 's and
 Π_r 's, respectively

\hat{m} = rank of dimensional matrix
= m (i.e., number of dimensions) usually

Dimensional Analysis

Methods for determining Π_i 's

1. Functional Relationship Method

Identify functional relationships $F(A_i)$ and $f(\Pi_j)$ by first determining A_i 's and then evaluating Π_j 's

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|------------------------|-----------|
| a. Inspection | intuition |
| b. Step-by-step Method | text |
| c. Exponent Method | class |

2. Nondimensionalize governing differential equations and initial and boundary conditions

Select appropriate quantities for nondimensionalizing the GDE, IC, and BC e.g. for M, L, and t

Put GDE, IC, and BC in nondimensional form

Identify Π_j 's

Exponent Method for Determining Π_j 's

- 1) determine the n essential quantities
- 2) select \hat{m} of the A quantities, with different dimensions, that contain among them the \hat{m} dimensions, and use them as repeating variables together with one of the other A quantities to determine each Π .