AVL Trees

In order to have a worst case running time for insert and delete operations to be O(log n), we must make it impossible for there to be a very long path in the binary search tree. The first balanced binary tree is the AVL tree, named after it's inventors, Adelson-Velskii and Landis. A binary search tree is an AVL tree iff each node in the tree satisfies the following property:

The height of the left subtree can differ from the height of the right subtree by at most 1.

Based on this property, we can show that the height of an AVL tree is logarithmic with respect to the number of nodes stored in the tree.

In particular, for an AVL tree of height H, we find that it must contain at least F_{H+3} -1 nodes. (F_i is the ith Fibonacci number.) To prove this, notice that the number of nodes in an AVL tree is the 1 plus the number of notes in the left subtree plus the number of nodes in the right subtree. If we let S_H represent the minimum number of nodes in an AVL tree with height H, we get the following recurrence relation:

$$S_H = S_{H-1} + S_{H-2} + 1$$

We also know that S_0=1 and S_1=2. Now we can prove the assertion above through induction.

For those of you who haven't seen induction yet, I won't "test" on it in this class. I'll try to explain the major steps of induction as best as I can, very briefly. Induction is used to prove that some statement or formula is true for all positive integers. Sometimes, it is difficult to prove a formula for all positive integers outright though.

In these cases, it may be easier to prove that IF the formula is true for an integer, say, 10 (we can call this k), then it MUST BE true for the next integer 11 (this would be k+1).

Finally, if we can prove that, AND we can show that the formula is true when you plug in 1 into it, it follows that the formula is true for all positive integers.

Here's a simple example you can hopefully relate to (I apologize to women who have small wardrobes!!!):

Assumptions: A female's wardrobe increases by 15% a year. A male's wardrobe increases by 10% a year. At the age of 20, a female has 50 pieces of clothing, while a male has 45.

We will prove: That for all years over 20 years of age, females own more pieces of clothing than males.

It is true for age 20 based on the given information.
Assume it's true for age k, where k □ 20.