EE468G NOTES (5C)

Reading assignment: Chapter 8 and Chapter 9

Contents: Magnetic material and inductance

HW 5e: Chapter 8: 4, 6, 7
Due: 12:30pm, Thursday, October 9.

HW 5f: Chapter 9: 1, 2, 3, 11, 12, 13
Due: 3:00pm, Monday, October 13.

Objectives:

(1) Magnetic material and permeability
(2) Boundary condition of magnetic fields
(3) Faraday's Law of inductance
(4) Calculation of self and mutual inductances

TEST 2: 12:30pm – 1:45pm,
Tuesday, October 14, 2003
Closed book

Magnetic material and boundary condition

Rotation of electron around nucleus form magnetic dipole $\overline{M}_k$. The total magnetic dipole per unit volume is defined as the magnetization vector:

Magnetization vector: $$\bar{M} = \lim_{\Delta v \to 0}\left(\frac{\sum_k \vec{M}_k}{\Delta v}\right), \qquad [A/m]$$

The magnetic potential produced by the magnetization vector is

$$\vec{A}_m = \frac{\mu_0}{4\pi}\int_V \frac{\nabla' \times \vec{M}}{R} dV' + \frac{\mu_0}{4\pi}\int_S \frac{\vec{M} \times \hat{a}_n}{R} dS'$$

Meanwhile, the magnetic potential produced by the imposed volume current and surface current is given by

$$\vec{A}_i = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}}{R} dV' + \frac{\mu_0}{4\pi}\int_S \frac{\vec{J}_s}{R} dS'$$

We conclude that the effect of magnetization vector is equivalent to an induced volume and surface current:

$$\vec{J}_M = \nabla \times \vec{M}$$
$$\vec{J}_{SM} = \vec{M} \times \hat{a}_n,$$

$\hat{a}_n$ is the outward normal direction of S.

Using the induced currents in the Ampere's law, we have

$$\nabla \times \vec{B} = \mu_0 \vec{J}_{\text{total}} = \mu_0 \left(\vec{J} + \vec{J}_M\right) = \mu_0 \vec{J} + \mu_0 \nabla \times \vec{M}$$

This means: $\nabla \times \left(\vec{B} / \mu_0 - \vec{M}\right) = \vec{J}$

Comparing this equation with Maxwell equation: $\nabla \times \vec{H} = \vec{J}$, we established that

$$\vec{H} = \vec{B} / \mu_0 - \vec{M}, \qquad \text{or} \qquad \vec{B} = \mu_0 \vec{H} + \vec{M}$$

Simple magnetic material (isotropic), $\vec{M}$ is parallel to $\vec{H}$,

$$\vec{M} = \mu_0 \chi_m \vec{H} \quad \Rightarrow \quad \vec{B} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H} = \mu \vec{H}$$

$$\mu = \mu_0 (1 + \chi_m)$$

Where, we define relative permeability as: $\mu_r = 1 + \chi_m$

Example: A material has $\mu_r = 2000$ and atom density of 2×10^{27} [m^{-3}]. If it is put in a uniform magnetizing field with $B_m = 0.15$ [T], (1) calculate $\vec{M}$ and the magnetic moments per atom.

Solution: (1) From $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$, we have

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \frac{\vec{B}}{\mu_0} - \frac{\vec{B}}{\mu_0 \mu_r} = \frac{1}{\mu_0}\left(1 - \frac{1}{\mu_r}\right)\vec{B}$$

M is in the same direction of B, hence

$$M = \frac{1}{4\pi \times 10^{-7}}\left(1 - \frac{1}{2000}\right) \times 0.15 = 0.12 \times 10^{6} \text{ [A/m]}$$

(2) Moment per atom is: $m = \dfrac{0.12 \times 10^{6}}{2 \times 10^{27}} = 0.06 \times 10^{-21}$ [A-m]