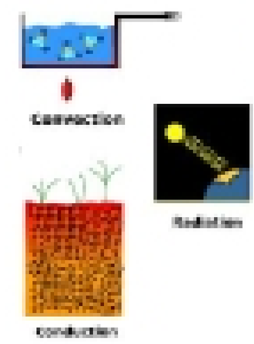
Lecture Ch. 3a

- Types of transfers
- Radiative transfer and quantum mechanics
  - Kirchoff's law
  - Blackbody radiation
  - Planck's radiation law
  - Wien's displacement law
  - Stefan-Boltzmann law

Curry and Webster, Ch. 3 pp. 74-85
For Thursday: Read Ch. 3 and Ch. 12 pp. 331-337
For Tuesday, 10/14: **Homework Ch. 3, pp.94-95:#1,2,3;** Read Ch. 4

What are the 3 ways heat can be transferred?



- **Radiation**: transfer by electromagnetic waves.
- **Conduction**: transfer by molecular collisions.
- **Convection**: transfer by circulation of a fluid.

Curry and Webster:
- Energy
  - Radiation
  - Conduction
  - Advection
- Scalars
  - Diffusion
  - Advection

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Scalar Transport

- Mass conservation
  - A continuity equation expresses a conservation law by equating a net flux over a surface with a loss or gain of material within the surface.
  - Continuity equations often can be expressed in either integral or differential form.

The conservation of mass is expressed by the continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0 \qquad (3.7)$$

- Transport

$$\frac{\partial C}{\partial t} + u_j \frac{\partial C}{\partial x_j} = \frac{1}{\rho} S_C \qquad (3.11)$$

Energy Transport

- Thermodynamic changes with time

The time variation of temperature can be written from (2.18b) as

$$c_v \frac{dT}{dt} = \frac{dq}{dt} + p \frac{d\alpha}{dt} \qquad (3.1)$$

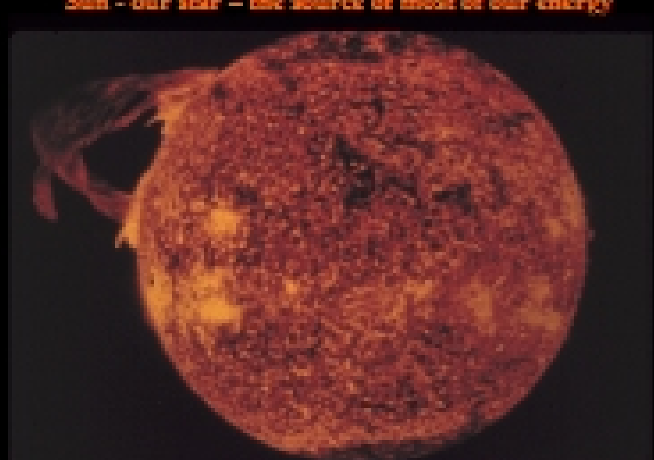
Using the definition of potential temperature (2.63) for the atmosphere or (2.73) and (2.34) for the ocean, (3.1) becomes

$$c_p \frac{T}{\theta} \frac{d\theta}{dt} = \frac{dq}{dt} \qquad (3.2)$$

- Thermodynamic changes with transport

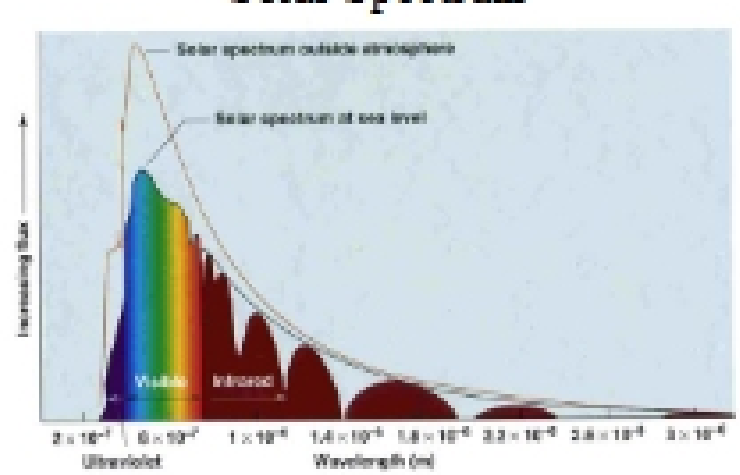
$$\frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} = \frac{1}{c_p} \frac{\theta}{T} \frac{dq}{dt} \qquad (3.6)$$

Sun - our star – the source of most of our energy



For the entire earth, climate can be explained by: 1) the amount of sunlight received and 2) the character of the surface receiving it.

Solar Spectrum



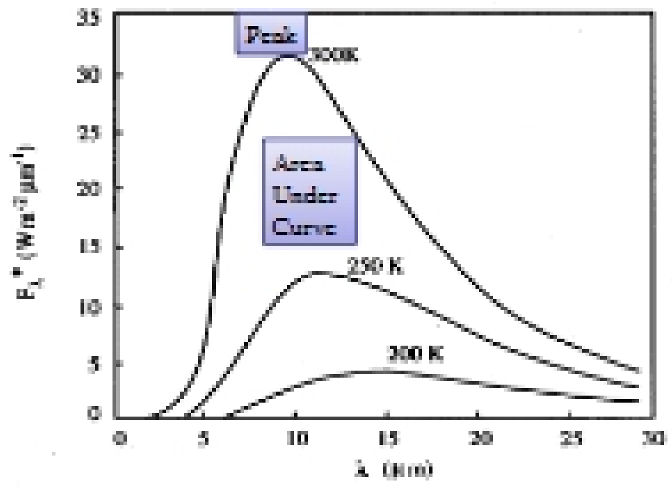


Figure 3.1 Black-body irradiance curves for terrestrial temperatures.

Planck's Radiation Law

- Direct consequence of quantum theory

The theory of black-body radiation was developed by Planck in 1900. Planck determined a semi-empirical relationship that included the concept that energy is quantized. Planck showed from quantum theory that the black-body irradiance, F_λ^*, is given by

$$F_\lambda^* = \frac{2\pi h c^2}{\lambda^5\left[\exp\left(\frac{hc}{k\lambda T}\right)-1\right]} \qquad (3.19)$$

where h is *Planck's constant* and k is Boltzmann's constant. Equation (3.19) is known as *Planck's radiation law*.

Solar Radiation

- Luminosity of the sun $L_0 = 3.9\times10^{26}$ W (p. 221)
- Irradiance F=Luminosity/Area=$L_0/(4\pi r^2)$= 6.44×10^7 W/m^2
  - r_{sun}=6.96×10^8 m [p. 427]
- Estimate blackbody radiation $T_{BB,sun}$=$(F/\sigma)^{1/4}$= 5800K
  - σ=5.67×10^{-8} W m^{-2} K^{-4} [p. 427]

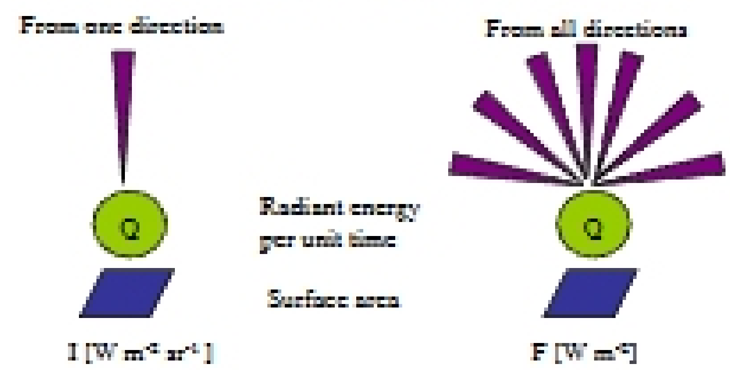
$$F^* = \int_0^\infty F_\lambda^*\, d\lambda = \sigma T^4 \qquad (3.20)$$

- Use Wien's law to evaluate λ_{max} = 0.5 μm (visible)

$$\lambda_{max} = \frac{2897.8}{T} \qquad (3.21)$$

- Similarly, λ_{earth} = 10 μm (infrared) for T_{earth} = 300K

Radiance and Irradiance



$$F = \int_0^{2\pi} I \cos Z\, d\omega \qquad (3.12)$$

Wavelength Dependence

Since the radiant energy is distributed over a spectrum of wavelengths, we define monochromatic radiance, I_λ, and irradiance, F_λ, as

$$I = \int_0^\infty I_\lambda\, d\lambda \quad \text{and} \quad F = \int_0^\infty F_\lambda\, d\lambda \qquad (3.14)$$

- Shortwave
  - Solar
  - Wavelengths 0.2–4 μm
- Longwave
  - Terrestrial
  - Wavelengths 4–200 μm

Wien's Displacement Law

- Inverse dependence of wavelength on temperature

The wavelength of maximum emission for a black body is found by differentiating Planck's law (3.19) with respect to the wavelength, equating to zero, and solving for the wavelength. This yields *Wien's displacement law*

$$\lambda_{max} = \frac{2897.8}{T}$$

This is the location of the peak!

where T is in K and λ_{max} is in μm. Evaluation of Wien's displacement law at T = 6000 K and T = 300 K shows that $\lambda_{max}(6000)$ = 0.48 μm and $\lambda_{max}(300)$ = 9.66 μm. Thus the wavelength of peak emission from the sun lies in the visible portion of the electromagnetic spectrum, while that from the Earth lies in the infrared.

Radiation Laws - Wien's Displacement Law

- Although all known objects emit all forms of electromagnetic radiation, the **wavelength of most intense radiation is inversely proportional to the T.** (1/T)

- Implications:
  - Sun emits @ ~ 6000 deg Kelvin
  - Earth emits @ 288 deg Kelvin,

- Which will emit radiation at the longer wavelength?
  - Earth

- The peak of Solar output is in the visible (light, shorter) part of the electromagnetic spectrum while the Earth, emits most of its energy in the infrared (heat, longer) portion of the electromagnetic spectrum

Radiation Laws - Wien's displacement law

- What does this mean in terms of the Earth and the Sun?

- **Warm objects,** Sun (6000°K) emit peak radiation at relatively short wavelengths (0.5 micrometers (1 millionth of a meter) – yellow-green visible)

- **Colder objects** Earth-atmosphere (average T of 288 °K, 15°C, 59°F) emit peak radiation at longer wavelengths (10 microns – infrared part of the spectrum)

- Most of the **sun's energy is emitted in a spectrum from 0.15 μm to 4 μm. 41% of it is visible, 9% is uv, 50 % infra-red.**

- **Earths radiant energy, stretches from 4 to 100μm, with maximum energy falling at about 10.1 μm (infrared).**

Stefan-Boltzmann Law

- Describes T^4 dependence of emission

Integration of (3.19) over all wavelengths gives

This is the area under the curve!

$$F^* = \int_0^\infty F^*_\lambda \, d\lambda = \sigma T^4 \qquad (3.20)$$

where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is called the *Stefan–Boltzmann constant*. Equation (3.20) is referred to as the *Stefan–Boltzmann law*, whereby the irradiance emitted by a black body varies as the fourth power of the absolute temperature. Evaluation of the Stefan–Boltzmann law at $T = 6000$ K (the approximate emission temperature of the sun) and $T = 300$ K (the approximate emission temperature of the Earth's surface) shows that $F^*(6000) = 7.35 \times 10^7$ W m^{-2} and $F^*(300) = 4.59 \times 10^2$ W m^{-2}, a difference of five orders of magnitude.

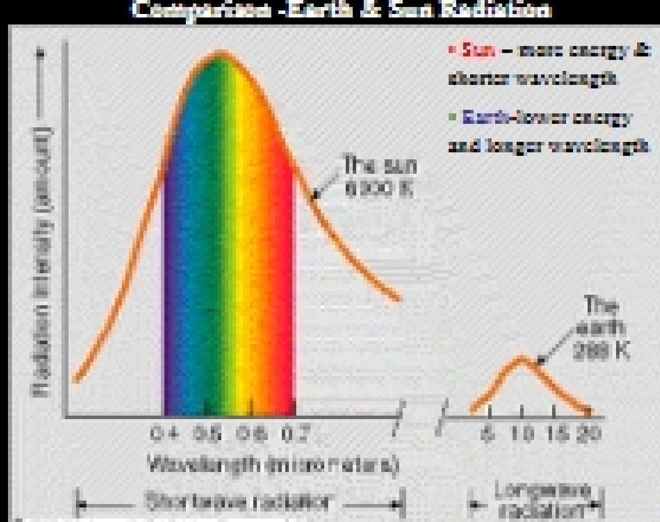
Blackbody Radiation

- Maximum possible emission of radiation

If a body emits the maximum amount of radiation at a particular temperature and wavelength, or equivalently absorbs all of the incident radiation, it is called a *black body*. For a black body, $A_\lambda = 1$ and $\Re_\lambda = T_\lambda = 0$ for all wavelengths. *Black-body radiation* is characterized by the following properties:

1. The radiant energy is determined uniquely by the temperature of the emitting body.
2. The radiant energy emitted is the maximum possible at all wavelengths for a given temperature.
3. The radiant energy emitted is isotropic.

Comparison -Earth & Sun Radiation



Radiation Laws - Black Body Radiation

- **Several physical laws describe the properties of electromagnetic radiation that is emitted by a perfect radiator, a so-called** black body.

- **By definition, at a given temperature, a black body absorbs all radiation incident on it at every wavelength and emits all radiation at every wavelength at the maximum rate possible for a given temperature;**

- **No radiation is reflected.**

- **A blackbody is therefore a perfect absorber and a perfect emitter.**