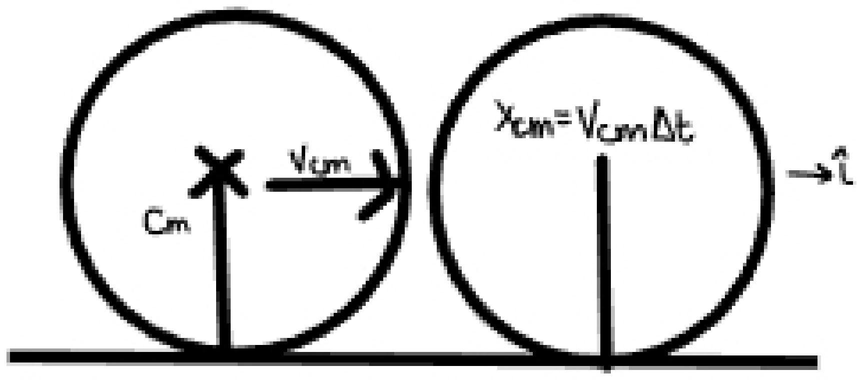
Lecture 25

Translational motion: the gravitational external force acts on the center of mass
Rotational motion: object rotates about center of mass
Note that the mass may be accelerating

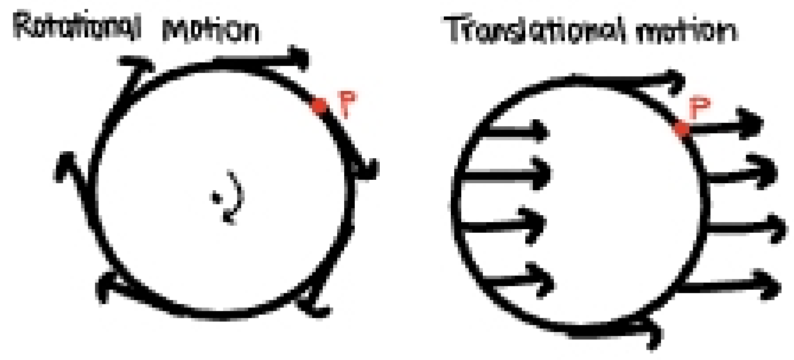


Translational Motion

Fixed Axis Rotation and Translation
Example: For straight line motion, the bicycle wheels rotate about a fixed direction and the center of mass translates

1. $\Delta s > \Delta x_{cm}$ slipping
2. $\Delta s = \Delta x_{cm}$ rolling without slipping
3. $\Delta s < \Delta x_{cm}$ skidding

Rolling without slipping
$\Delta s = \Delta x_{cm} \longrightarrow R\Delta\theta = v_{cm}\Delta t \longrightarrow v_{cm} = R\frac{\Delta\theta}{\Delta t} \longrightarrow v_{cm} = \pm R\omega_z$
The sign depends on the choice on linear coordinate system and rotational coordinate system



$\vec{V}_P = \vec{V}_{rot} + \vec{V}_{tran}$

Velocity Vectors

Translational Motion of Center of Mass: Newton's Second and Third Laws
The momentum of a system remains constant unless the system is acted on by an external force in which case the acceleration of center of mass satisfies
$F_{ext} = m_{total}\vec{a}_{cm}$
Impulse changes center of mass momentum
$I = m_{total}(\vec{v}_{cm}(t_f) - \vec{v}_{cm}(t_0))$

Fixed axis rotation about the center of mass of a rigid body
The total external torque produces an angular acceleration about the center of mass $\vec{\tau}^{ext}_{cm} = I_{cm}\vec{\alpha}_{cm} = \frac{d\vec{L}_{cm}}{dt}$
I_{cm} is the moment of inertial about the center of mass
α_{cm} is the angular acceleration about the center of mass
L_{cm} is the angular momentum about the center of mass

Angular Momentum for two dimensional rotation and translation
The angular momentum for a translating and rotating object is given by
$\vec{L}_s = \vec{R}_{s,cm} \times \vec{p}_{sys} + \sum \vec{r}_{cm,i} \times m_i\vec{v}_{cm,i}$
Angular momentum arising from translational of center of mass
$\vec{L}_{s,cm} = \vec{R}_{s,cm} \times \vec{p}_{sys}$
The second term is the angular momentum arising from rotation about the center of mass
$\vec{L}_{cm} = \sum \vec{r}_{cm,i} \times m_i\vec{v}_{cm,i} = I_{cm}\vec{\omega}_{cm}$

Kinetic Energy of Rotation and Translation
Kinetic energy of rotation about center of mass
$K_{rot} = \frac{1}{2} I_{cm}\omega^2_{cm}$
Translational kinetic energy
$K_{trans} = \frac{1}{2} m v^2_{cm}$
Kinetic
$K = K_{trans} + K_{rot} = \frac{1}{2} m v^2_{cm} + \frac{1}{2} I_{cm}\omega^2_{cm}$

Rotational and Translational Motion

Quantity	Fixed Axis Rotation	Translation
Momentum (mag.)		$p = mV_{cm} = \sqrt{2mK_{trans}}$
Ang. Momentum (mag.)	$L_{cm} = I_{cm}\omega = \sqrt{2I_{cm}K_{rot}}$	
Force		$\vec{F}_{ext} = d\vec{p}_{sys}/dt = m_{total}\vec{A}_{cm}$
Torque	$\vec{\tau}_{cm} = d\vec{L}_{cm}/dt = I_{cm}\vec{\alpha}$	
Kinetic Energy	$K_{rot} = (1/2)I_{cm}\omega^2$ $K_{rot} = L^2_{cm}/2I_{cm}$	$K_{trans} = (1/2)mV^2_{cm}$ $K_{trans} = p^2/2m$
Work	$W = \int_{\theta_i}^{\theta_f} \tau_z\, d\theta$	$W = \int \vec{F} \cdot d\vec{r}$
Power	$P_{rot} = \tau_z\omega$	$P = \vec{F} \cdot \vec{v}$