Lecture 29

Bicycle wheel

Case one: non-spinning wheel

Magnitude of the angular momentum about the pivot changes. Bicycle wheel falls. Direction of change of angular momentum about pivot is the same as the direction of angular momentum about the pivot

Case two: spinning wheel $\vec{\tau}_S = \frac{d\vec{L}_S}{dt}$

Direction of angular momentum about the pivot changes. Bicycle wheel precesses about the vertical axis

Concept question: Rotating Vector

A vector A(t) of fixed length A is rotating about the z-axis at an angular speed ω. At t = 0 it is pointing in the positive y-direction. A(t) is given by:

$A(t) = -A\sin(\omega t)\hat{i} + A\cos(\omega t)\hat{j}$ and $\delta A(t)/\delta t = -\omega A\cos(\omega t)\hat{i} - \omega A\sin(\omega t)$

Torque and time derivative of angular momentum

Torque about S is equal to the time derivative of the angular momentum about S $\vec{\tau}_S^{ext} = \frac{d\vec{L}_S}{dt}$

If the magnitude of the angular momentum is constant, then the torque can cause the direction of the perpendicular component of the angular momentum to change $|\vec{\tau}_S^{ext}| = |\vec{L}_{S,\perp}| |\frac{d\theta}{dt}|$

Gyroscopic approximation

Flywheel is spinning with an angular velocity $\vec{\omega} = \omega\hat{r}$

Precessional angular velocity: $\vec{\Omega} = \Omega\hat{k}$

Gyroscopic approximation: the angular velocity of precession is much less than the component of the spin angular velocity

Torque on a gyroscope: change in angular momentum

Torque about S changes direction of angular momentum leaving magnitude unchanged. Gyroscope precesses, does not fall

Gyroscope: Time derivative of angular momentum

If the angular speed (precession angular speed) about the z-axis is constant then only the direction of the spring angular momentum $\vec{L}_S^{spin} = I_s\omega_s\hat{r}$ along the axis of the gyroscope is changing in time hence $\frac{d\vec{L}_S}{dt} = I_s\omega_s\Omega_z\hat{\theta}$

Torque and time derivative of angular momentum

Torque about S: $\vec{\tau}_S^{ext} = dmg\hat{\theta}$

Torque Law: Toque is equal to the time derivative of the angular momentum about S $\vec{\tau}_S = \frac{d\vec{L}_S}{dt}$

Therefore $dmg\hat{\theta} = I_s\omega_s\Omega_z\hat{\theta}$

Precession angular velocity is $\vec{\Omega} = \Omega\hat{k} = \frac{dmg}{I_s\omega_s}$