Math/Stat 370: Engineering Statistics

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Week 10

Sample Pairs from Two Populations

- Two populations with, respectively, unknown means μ_1 and μ_2, unknown variances σ_1^2 and σ_2^2.
- Hypotheses: $H_0 : \mu_1 - \mu_2 = \Delta_0$ VS $H_1 : \mu_1 - \mu_2 \neq \Delta_0$.
- Test the hypotheses with significant level α.
- Take sample pairs $(X_{11}, X_{21}), (X_{12}, X_{22}), \ldots, (X_{1n}, X_{2n})$ from two populations.
- Within each pair, two random variables may be dependent.

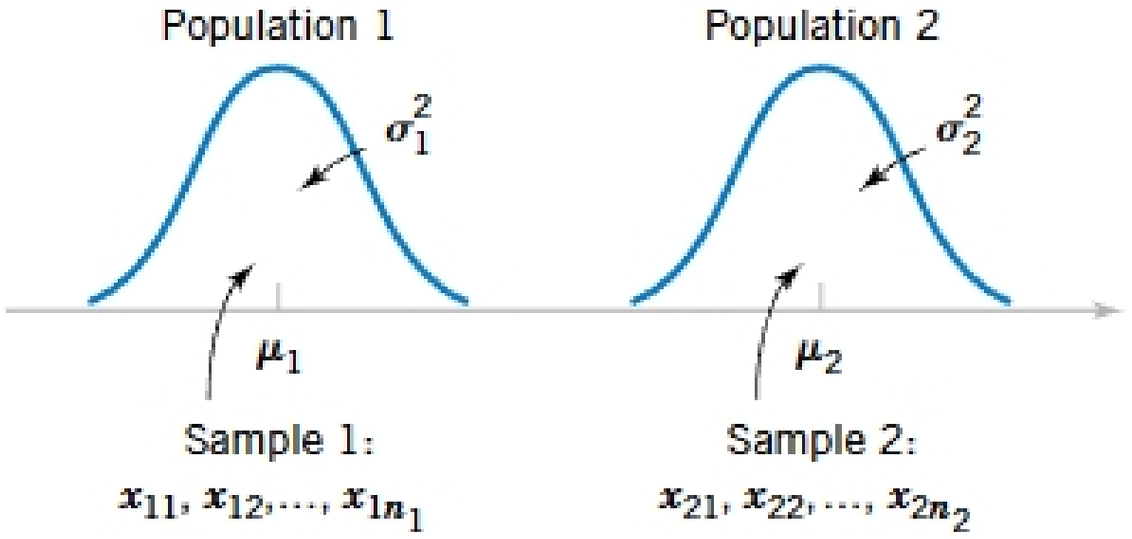

Figure: Same sample size: $n_1 = n_2$.

Average of Sample Difference

- The sample average of the first sample

$$\overline{X}_1 = \frac{\sum_{i=1}^{n} X_{1i}}{n}.$$

- The sample average of the second sample

$$\overline{X}_2 = \frac{\sum_{i=1}^{n} X_{2i}}{n}.$$

- Let $\overline{D} = \overline{X}_1 - \overline{X}_2 = \frac{1}{n}\sum_{i=1}^{n}(X_{1i} - X_{2i})$.
- $E(\overline{D}) = E(\overline{X}_1 - \overline{X}_2) = \mu_1 - \mu_2$.
- $V(\overline{D}) \leq V(\overline{X}_1) + V(\overline{X}_2) = \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{n}$.