

**Econ 311: Labor Supply and the Two-Step  
Estimator  
Revised Version**

One period models

$$\begin{aligned} \max U(c, l) &= \left( \frac{c^\alpha - 1}{\alpha} \right) + b \left( \frac{l^\phi - 1}{\phi} \right) \\ \text{st. } c + wl &\leq wT + A. \end{aligned}$$

The Euler equation is  $w = \frac{bl^{\phi-1}}{c^{\alpha-1}}$  and the

reservation wage is given by

$$\begin{aligned}w_r &= \left[ \frac{bl^{\phi-1}}{c^{\alpha-1}} \right]_{l=1, c=A} = \frac{b}{A^{\alpha-1}} \rightarrow \ln w_r \\ &= \ln b + (1 - \alpha) \ln A\end{aligned}$$

Assume  $\ln b = x\beta + e$ ,  $e \perp\!\!\!\perp (x, A, w)$ ,  $e$

$\sim N(0, \sigma_e^2)$ . Assume wages observed for

everyone

$$\begin{aligned}
& \Pr(\text{Person works} | X, A) \\
&= \Pr(\ln w_r \leq \ln w | x, A) \\
&= \Pr\left(\frac{e}{\sigma_\epsilon} \leq \frac{\ln w - x\beta - (1 - \alpha) \ln A}{\sigma_\epsilon}\right) \\
&= \Phi(c)
\end{aligned}$$

$$\text{where } c = \frac{\ln w - x\beta - (1 - \alpha) \ln A}{\sigma_\epsilon}.$$

### **Grouped data estimator**

Each cell has common values of

$w_i, x_i, A_i$ . For each cell, obtain  $\hat{P}_i(D_i =$

$1 | w_i, x_i, A_i) = \text{cell proportion working}$