continue solving using intergration by parts for:

$2/9 \int x e^{9x}\,dx = 2/9 \frac{x}{9} \cdot e^{9x} - 2/9 \int 1 \cdot 1/9\, e^{9x}\,dx = \to$

Solution: $u(x) = x \quad V'(x) = e^{9x}$

$u'(x) = 1 \quad V(x) = 1/9\, e^{9x}$

$\to = \frac{2x}{81} e^{9x} - 2/9 \cdot 1/9 \int e^{9x}\,dx = \frac{2x}{81} e^{9x} - \frac{2}{81} \cdot 1/9\, e^{9x} + C =$

$\to = \frac{2x}{81} e^{9x} - \frac{2}{729} e^{9x} + C \Rightarrow \frac{x^2}{9} e^{9x} - \left(\frac{2x}{81} e^{9x} - \frac{2}{729} e^{9x} + C\right) =$

$\to = \boxed{\frac{x^2}{9} e^{9x} - \frac{2x}{81} e^{9x} + \frac{2}{729} e^{9x} + C_1}$, C is a constant

③ a.) evaluate $\int x^3 \ln x^4\,dx$ by substitution using $u = x^4$, then evaluating using remaining intergral by using intergration by parts

Solution: $\int x^3 \ln x^4\,dx = \int \cancel{x^3} \ln u \frac{du}{4\cancel{x^3}} = \int \ln u \frac{du}{4} = 1/4 \int \ln u\,du$

1.) substitution $u = x^4$

2.) $\frac{du}{4x^3} = \frac{4x^3\,dx}{4x^3} = dx = \frac{du}{4x^3}$ Formula: $\int u\,dv = \overset{w}{u}v - \int v\,du$

Intergration by parts

$\frac{1}{4} \int \ln u\,du = \frac{1}{4} u \ln u - \frac{1}{4} \int u \cancel{\frac{1}{u}}\,du = 1/4\, u \cdot \ln u - 1/4 \int 1\,du = \to$

1.) choose $w = \ln u$ & $dv = du$ $\quad w = \ln u\,du = \frac{1}{u} \cdot du = du/u$

$dw = \frac{du}{u} \quad v = u$

$\to = \frac{1}{4} \cdot u \cdot \ln u - 1/4 \cdot u (4/4) + C = \boxed{\frac{1}{4} \cdot x^4 \cdot \ln x^4 - \frac{x^4}{4} + C}$

b.) Evaluate $\int x^3 \ln x^4\,dx$ by using intergration by parts

Lecture: 29-9-14 Professor Shabanskaya Calculus 1760-002

Section 7.3: Rational Functions & Partial Fractions.

$\int \frac{P(x)}{Q(x)}\,dx$, where $\frac{P(x)}{Q(x)}$ is a rational function ie:

$P(x) = a_n x^n + a_{n-1} + \cdots + a_1 x + a_0$, $Q(x) = b_n x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0$ are polynomials with degree $(P(x)) = n$ & degree $(Q(x)) = m$.

highest power of x

If degree $(P(x)) \geq$ degree $(Q(x))$, then we divide $P(x)$ by $Q(x)$ applying long division & obtain

$\frac{P(x)}{Q(x)} = K(x) + \frac{P_1(Q)}{Q(x)}$, where degree $P_1(x) \leq$ degree $(Q(x))$

polynomial

Assume without ~~[illegible]~~ loss of generality that degree $(P(x)) <$ degree $(Q(x))$ in $\frac{P(x)}{Q(x)}$. We have the following cases for $\frac{P(x)}{Q(x)}$

1.) $\frac{~~p(x)~~\quad P(x)}{a(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)} = \frac{1}{a}\left[\frac{A_1}{x-x_1}+\frac{A_2}{x-x_2}+\frac{A_3}{x-x_3}+\cdots+\frac{A_n}{x-x_n}\right]$

2.) $\frac{p(x)}{a(x-b)^2} = \frac{1}{a}\left[\frac{B_1}{x-b}+\frac{B_2}{(x-b)^2}+\frac{B_3}{(x-b)^3}+\cdots\frac{B_n}{(x-b)^n}\right]$

→ $x$ is a variable, $x_1 - x_n$ are numbers, $A_1, A_2, A_3, \cdots A_n$ are constants.

→ $x$ is a variable, $b$ is a constant as well as $B_1, B_2, B_3, \cdots B_n$ are constants.

Ex: Use partial fraction decomposition to evaluate each of the integrals (p. 351)

33. $\int \frac{x^2+1}{x^2+3x+2}\,dx$ (is rational function)

1.) Degree $(x^2+1) = 2$
Degree $(x^2+3x+2) = 2$ (Since degrees are equal, apply long division)

2.)
$$\begin{array}{r|l} & 1 \\ \hline x^2+3x+2 & x^2+1 \\ & -\underline{(x^2+3x+2)} \\ & -3x-1 \end{array}$$

(We stop doing long division when remainder is less than what you are dividing by)

$-3x-1$ degree $(-3x-1) <$ degree $(x^2+3x+2)$

$$\boxed{\frac{x^2+1}{x^2+3x+2} = 1 + \frac{-3x-1}{x^2+3x+2}}$$

3.) Now degree $(-3x-1) <$ degree $(x^2+3x+2)$ in $\frac{-3x-1}{x^2+3x+2}$

$\int \frac{x^2+1}{x^2+3x+2}\,dx = \int 1\,dx + \int \frac{-3x-1}{x^2+3x+2}\,dx = x + \int \frac{-3x-1}{x^2+3x+2}\,dx$

4.) Factor $x^2+3x+2$. (Not possible to factor if the discriminant $D = b^2-4ac < 0$ of $ax^2+bx+c = 0$.

$(x+1)(x+2) = x^2+3x+2$ means we can rewrite: $\frac{-3x-1}{x^2+3x+2}$ as: $\frac{A_1}{x+1} + \frac{A_2}{x+2}$. $A_1/x+1$ has to be multiplied & divided by $x+2$. $A_2/x+2$ has to be multiplied & divided by $x+1 = \frac{A_1(x+2)+A_2(x+1)}{(x+1)(x+2)}$.

Since denominators are the same, numerators are the same ie: $-3x-1 = A_1(x+2) + A_2(x+1)$

$A_1x + 2A_1 + A_2x + A_2$ (combine and get) $= -3x-1 = (A_1+A_2)x + (2A_1+A_2)$

$A_1 + A_2 = -3$ $(2A_1+A_2) = -1 = -A_1 = -2$ in ex: $\boxed{A_1 = 2}$

$2 + A_2 = -3 = \boxed{A_2 = -5}$

$\int \frac{-3x-1}{x^2+3x+2} = \int \frac{2}{x+1}\,dx + \int \frac{-5}{x+2}\,dx$