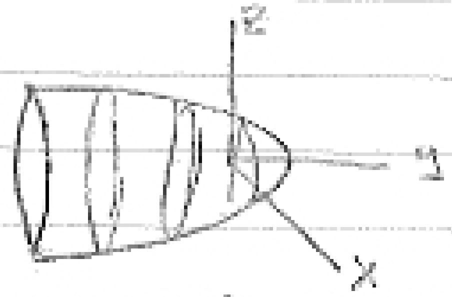
$T = \frac{v(t)}{|v(t)|}$ $N = \frac{\frac{dT}{dt}}{\left|\frac{dT}{dt}\right|}$ $B = \frac{v \times a}{|v \times a|}$ $N = B \times T$ $\kappa = \frac{|v \times a|}{|v(t)|^3} = \frac{\left|\frac{dT}{dt}\right|}{v(t)}$

$z = 0: 4 = x^2 + y$

$y = 4 - x^2$




graph

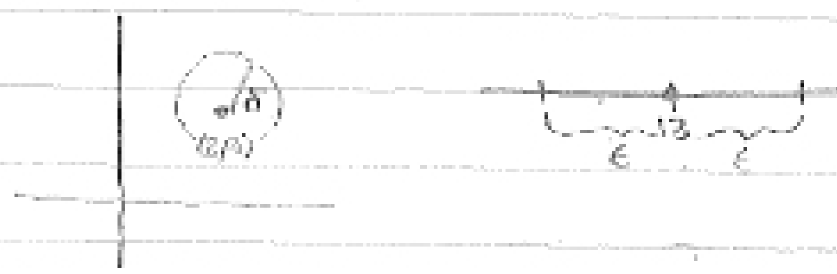
9/11/14

*EXAM TOMORROW! CHAPTER 13*

limits and continuity

$\lim_{x \to 3} f(x) = 7$ means when $x$ is "close" to 3 then $f(x)$ is "close" to 7.

$\lim_{(x,y) \to 8,9} f(x,y) = 13$




f is continuous at $x = x_0$ if:

1) $f(x_0)$ exists
2) $\lim_{x \to x_0} f(x)$ exists
3) 1 & 2 equal each other.

$f(x) = \frac{x^3 + x^2 + 2x}{x^6 + x^4 + 3x^2}$ Continuous except at $x = 0$.

$g(x) = \frac{\sqrt{x^2 - 4}}{x^2 + x^4}$ $x^2 - 4 \geq 0$

$x \geq 2$ & $x \leq -2$

$f(x) = \frac{x^3 + x + 5}{x^4 + x^2}$ $\lim_{x \to 1} f(x) = f(1) = \boxed{7/2}$

$T = \frac{r'(t)}{|r'(t)|}$ $N = \frac{\frac{dT}{dt}}{\left|\frac{dT}{dt}\right|}$ $B = \frac{r' \times r''}{|r' \times r''|}$ $\kappa = \frac{|r' \times r''|}{|r'(t)|^3}$

$$\lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y - xy + 4x^2 - 4x} = \frac{0}{0}$$

$\lim_{P \to P_0} f(P)$ but $f(P_0)$ doesn't exist

3 possibilities:

- do some algebra to write $f(P) = \frac{g(P)}{g(P)} h(P)$ where $h(P_0)$ is defined.
  - ie $\lim_{x\to 0} \frac{x^2}{x} = \lim_{x\to 0} \frac{x}{x} x = \lim_{x\to 0} x = 0$
- do a change of variables
  - ie use polar coordinates
- do $\epsilon - \delta$ arrangement

$$\lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y - xy + 4x^2 - 4x}$$

$$\frac{y+4}{x^2y - xy + 4x^2 - 4x} = \frac{\text{something}}{\text{something}} (\text{nice})$$

$$= \frac{y+4}{y+4}\left(\frac{1}{x^2 - x}\right)$$

$$\begin{array}{r|l} & x^2 - x \\ \hline y+4 & x^2y - xy + 4x^2 - 4x \\ & \underline{yx^2 - yx + 4x^2 - 4x} \\ & 0 \end{array}$$

$$\lim_{(x,y)\to(2,-4)} \frac{\cancel{y+4}}{\cancel{y+4}} \cdot \frac{1}{x^2 - x} = \lim_{(x,y)\to(2,-4)} \frac{1}{x^2 - x} = \frac{1}{4-2} = \boxed{\frac{1}{2}}$$

$K = \frac{|v \times a|}{|v(t)|^3}$ $\quad \tau = \frac{(v \times a) \cdot a'}{|v \times a|^2}$ $\quad T = \frac{v(t)}{|v(t)|}$ $\quad N = B \times T = \frac{T'}{|T'|}$ $\quad B = \frac{v \times a}{|v \times a|} = T \times N$

$$K = \frac{|v \times a|}{|v(t)|^3}$$

$$\lim_{(x,y)\to(4,3)} \frac{\sqrt{x} - \sqrt{y+1}}{x-y-1} \Rightarrow \frac{0}{0}$$

$$= \frac{\sqrt{x} - \sqrt{y+1}}{x-(y+1)}$$

$$x-(y+1) = \sqrt{x}^2 - \sqrt{y+1}^2$$
$$= (\sqrt{x} - \sqrt{y+1})(\sqrt{x} + \sqrt{y+1})$$

$$\lim_{(x,y)\to(4,3)} \frac{\cancel{\sqrt{x} - \sqrt{y+1}}}{(\cancel{\sqrt{x} - \sqrt{y+1}})(\sqrt{x} + \sqrt{y+1})} = \lim_{(x,y)\to(4,3)} \frac{1}{(\sqrt{x} + \sqrt{y+1})} = \boxed{\frac{1}{4}}$$

$$\lim_{(x,y)\to(0,0)} \frac{2x}{x^2+x+y^2}$$

*if going to 0,0 sometimes switch to polar pays off

switch to polar
$x = r\cos\theta$
$y = r\sin\theta$

$$\lim_{r\to 0} \frac{2r\cos\theta}{r^2\cos^2\theta + r\cos\theta + r^2\sin^2\theta} \Rightarrow \frac{2r\cos\theta}{r^2 + r\cos\theta} =$$

$$= \lim_{r\to 0} \frac{\cancel{r}\, 2\cos\theta}{\cancel{r}\,(r+\cos\theta)} = \boxed{2}$$