Math 19: Calculus **Winter 2008** **Instructor: Jennifer Kloke**

Lecture Outline (Derivative as a Function)
Monday, January 28

Announcements

1. Homework 2 is due Wednesday 01/30/08
2. There's review session for the midterm on Wednesday evening 7-9 in room 380W.
3. Midterm 1 is on Thursday 01/31/08 in Herrin 175.

Recap

On Friday we revisited the tangent line and computed lots of examples of it. We also talked about the two seemingly different expressions for the slope of the tangent line.

On Friday we also gave the definition of the derivative of a function at a point. We noted that the derivative $f'(a)$ is the slope of tangent line to $y = f(x)$ at $x = a$.

Derivative at a Point

Let's do one or two more examples with this concept before we move on to thinking about the derivative as a function.

(1) Sketch the graph of a function f for which $f(0) = 0$, $f'(0) = 3$, $f'(1) = 0$, and $f(2) = -1$.

Let's also connect these ideas back to the real world. Suppose we have a function $y = f(x)$ ($f(x)$ can be the price of producing x units, or it can be position of a particle at time x, or it can be concentration of a chemical in your blood a time x). The average rate of change of this function is given by:

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

We can do better than just the finding the average rate of change between two input values. We can find the instantaneous rate of change at $x = x_1$ by taking the limit of this expression:

$$= \lim_{x_2 \to x_1} \left(\frac{f(x_2) - f(x_1)}{x_2 - x_1} \right) = f'(x_1).$$

Look in your textbook, sections 2.6 and 2.7, for examples relating these ideas back to real world problems.

The Derivative as a Function

So far we've considered the derivative of a function *at a fixed point a*. Today, we're changing our perspective slightly and instead we'll speak of the derivative as a function itself. For a given input x, this derivative function will return for us the derivative of f at the point x. Explicitly

Definition: For a function $f(x)$, the derivative of $f(x)$, written $f'(x)$ or $\frac{d}{dx}(f(x))$, is defined to be

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

if it exists. The derivative $f'(x)$ is the function which gives the slope of the line tangent to f at the point x.

Note: this derivative function is very much like the limits we've been considering in the past few weeks. The only difference is taht instead of having an explicit value for x in the limit above, we leave this quantity a variable. This means that, generally speaking, the derivative $f'(x)$ will be a function of x and not just a number.

Examples: Derivative as a function

1. Use the definition of the derivative to compute $f'(x)$ where $f(x) = x^2$.

   We can use geometry to make computations a bit easier.

2. Using only that the derivative $f'(a)$ is the slope of the tangent line to f at a, find the derivative of $f(x) = c$.

   We could also compute this derivative using the definition. Let's try it out.

3. Using only that the derivative $f'(a)$ is the slope of the tangent line to f at a, find the derivative of $f(x) = mx + b$. Also compute thsi derivative using the definition above.

4. Here is the graph of $f(x) = \sin(x)$. Sketch the graph of $f'(x)$ on to the graph of $f(x)$.

What to know/memorize

Know the definition of the derivative of the function $f(x)$.
Know how to graph the function $f'(x)$ onto the graph of $f(x)$.