11/18/14

Vector field $\vec{F} = \langle M, N, P \rangle$

curve $C: \vec{r}(t) = \langle x(t), y(t), z(t) \rangle \quad a \le t \le b$

$$\int_C \vec{F} \cdot d\vec{r} = \underbrace{\int_C M\,dx + N\,dy + P\,dz}_{1\text{ form}}$$

$\boxed{\begin{array}{l}\text{Any } F \\ \int_{-C} F\,dr = -\int_C F\,dr\end{array}}$

F is <u>conservative</u> if

$\int_C F\,dr$ depends only on the endpoints & not C itself.

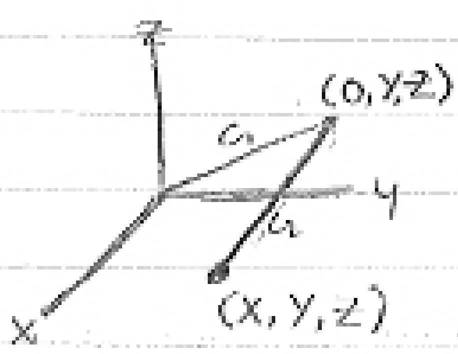
$M\,dx + N\,dy + P\,dz$ is <u>exact</u> in this case.

(conservative = exact)

$$f(XYZ) = \int M(xyz)\,dx + N(xyz)\,dy + P(xyz)\,dz$$

C: goes from base point (0,0,0) to (XYZ)

Want to compute $\frac{\partial f}{\partial X}$



$$f(XYZ) = \int_{C_1} M\,dx + N\,dy + P\,dz + \int_{C_2} M\,dx + N\,dy + P\,dz$$

$$\int_{C_1} M\,dx + N\,dy + P\,dz \Rightarrow \int_0^1 \cancel{M(0,Yt,Zt)}0 + N(0,Yt,Zt)Y\,dt + P(0,Yt,Zt)Z\,dt$$

$x = 0 \quad dx = 0$

$y = Yt \quad dy = Y\,dt$

$z = Zt \quad dz = Z\,dt$

$0 \le t \le 1$

That is independent of X so it goes away.

But this is what's left

$\int_{C_1} M\,dx + N\,dy + P\,dz$

$0 \le t \le X$

$x = t \quad dx = dt$

$y = Y \quad dy = 0$

$z = Z \quad dz = 0$

$\int_0^X M(t, Y, Z)\,dt + 0$

$\frac{\partial}{\partial X} \int_0^X M(t, Y, Z)\,dt$

$= M(X, Y, Z)$

If F is conservative then $F = \nabla f$ for some f
or if $M\,dx + N\,dy + P\,dz$ is exact then
$M\,dx + N\,dy + P\,dz = df$ for some f

EX) (from Yesterday)

$(2x + 4y)\,dx + (4x + 8y)\,dy$

constant is function of y b/c y is a constant

$f(xy) = \int (2x + y)\,\partial x = x^2 + xy + C(y)$

$\frac{\partial f}{\partial y} = 0 + x + C'(y) = 4x + 8y$

$C'(y) = 3x + 8y$

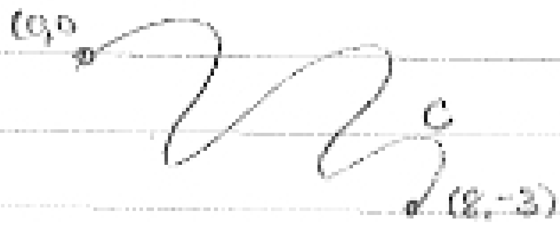
$C(y) = \int 3x + 8y\,dy$

$= 3xy + 4y^2 + C$

$f(xy) = x^2 + xy + 3xy + 4y^2$

$\boxed{f(xy) = x^2 + 4xy + 4y^2}$

More on next page.



Using previous page!

$$\int_c (2x+4y)dx + (4x+8y)dy = f(8,-3) - f(0,0)$$
$$= 64 - 96 + 36 - 0$$
$$= 4$$

Claim: $(8x-4y+12z)dx + (-4x+2y-6z)dy + (12x-6y+18z)dz$

$$f(xyz) = \int 8x - 4y + 12z \, \partial x$$
$$= 4x^2 - 4xy + 12xz + C(y,z)$$

$$\frac{\partial f}{\partial y} = 0 - 4x + 0 + \frac{\partial c}{\partial y} = -4x + 2y - 6z$$
$$\frac{\partial c}{\partial y} = 2y - 6z$$
$$c(yz) = \int (2y - 6z)dy$$
$$= y^2 - 6yz + C(z)$$

$$f(xyz) = 4x^2 - 4xy + 12xz + y^2 - 6yz + C(z)$$

$$\frac{\partial f}{\partial z} = 0 - 0 + 12x + 0 - 6y + C'(z) = 12x - 6y + 18z$$
$$C'(z) = 18z$$
$$\frac{\partial c}{\partial z} = \int 18z \, dz$$
$$C(z) = 9z^2$$

$$\boxed{f(xyz) = 4x^2 - 4xy + 12xz + y^2 - 6yz + 9z^2}$$

Compute:

$$\int_c \vec{F} dr = f(1,1,1) - f(0,0,0)$$

$C: (0,0,0) \rightarrow (1,1,1)$

$$= 16 - 0 = \boxed{16}$$