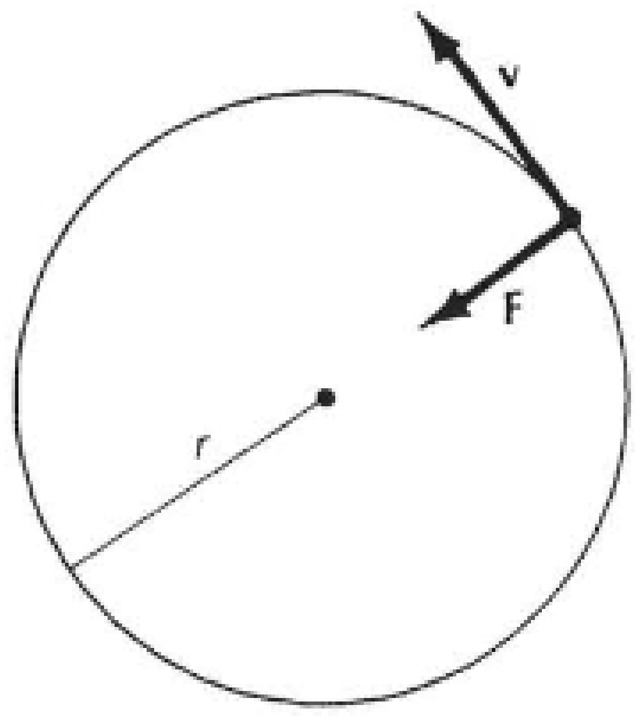
Dynamics of Uniform Circular Motion

For a particle moving with speed v in a circle of radius R, the centripetal acceleration is

$$a_c = \frac{v^2}{R}$$



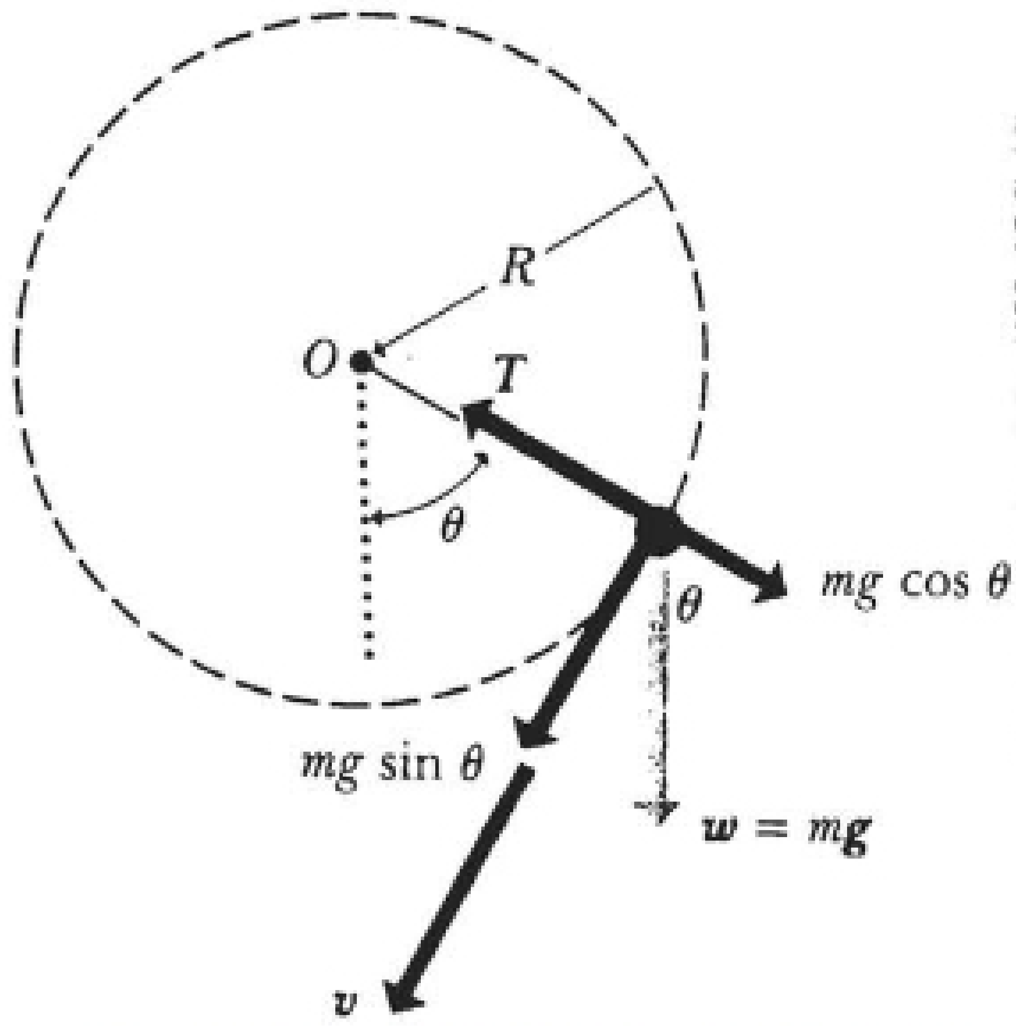
Centripetal force for a particle in uniform circular motion.

If the particle has a mass m, the required force is given by

$$F_c = m a_c = \frac{mv^2}{R}$$

Whenever a particle moves in a circle with a speed v, this force must be provided by some external agent.

Motion in a Vertical Circle.



Forces on a body whirling in a vertical circle with center at O.

- Ball whirled in a vertical circle about point O.
- Motion circular but is not uniform.
- Speed increases on the way down, decreases on the way up.
- v changes continuously around the path.
- ∴ We must have $a_\perp$ and $a_\parallel$
- Forces on ball are gravity and tension.

$$F_\parallel = mg \sin\theta \qquad ①$$

$$F_\perp = T - mg\cos\theta \qquad ②$$

The tangential acceleration:

$$a_\parallel = \frac{F_\parallel}{m} = g\sin\theta \qquad ③$$

$$a_\perp = \frac{F_\perp}{m} = \frac{T - mg\cos\theta}{m} = \frac{v^2}{R} \qquad ④$$

Solve ④ for $T = m\left(\frac{v^2}{R} + g\cos\theta\right)$

Lowest Point: $\theta = 0$

$F_{\parallel} = 0$, $a_{\parallel} = 0$, acceleration is purely radial.

$$T = m\left(\frac{v^2}{R} + g\right)$$

Highest Point: $\theta = 180^\circ$

- acceleration purely radial

$$T = m\left(\frac{v^2}{R} - g\right)$$

If speed equals a critical value v_c, tension vanishes, $T \equiv 0$.

$$0 = m\left(\frac{v_c^2}{R} - g\right)$$

$$v_c = \sqrt{Rg}.$$

The speed at any point can be determined from energy considerations given a value at some initial point.