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Appendix A

Scientific Notation, Units, and Constants

This appendix reviews the "scientific notation" used in the text; the units commonly used in physics and chemistry to quantify length, mass, and so on; and the manipulation of these units. We discuss some of the physical constants and the essential mathematical operations that are used in the text.

A.1 Scientific Notation

Most scientific discussions use numbers that are either very large or very small. Examples of such numbers can be found in every chapter of this book, including the total number of molecules in the Earth's atmosphere, 10^{44}; the fraction corresponding to a mixing ratio of one part per trillion by volume (pptv), 10^{-12}; the amount of carbon emitted into the atmosphere each year from fossil-fuel combustion, about 10^{19} grams; and the wavelength measure for ultraviolet radiation, roughly 10^{-7} meters. If we had to write out the first number, it would read: 100,000,000,000,000,000,000,000,000,000,000,000,000,000. That is, the digit 1 followed by a string of 44 zeros! The second number would read: 0.000000000001. These longhand numbers are clumsy and impractical.

Basically, any large or small number can be expressed as the product of two terms. The first term is a **prefactor** of order unity (that is, a number with a value between 1 and 10) that gives the precision, or accuracy, of the original number. The second term is a **power of 10** (that is, 10^p, where p is the **exponent** or "power" of 10). The power of 10 defines how many times the number 10 is to be multiplied by itself. For example, $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$ (that is, 10 multiplied by itself four times). As an example of the use of this notation, consider the large number 1,100,000 (which is named one million one hundred thousand). In scientific notation this number may be written 1.1×10^6.

Notice that to arrive at this form, we must count the digits from the position where the decimal point is originally found (or would be placed) in the number toward the left until only one non-zero digit remains to the left. The decimal point is fixed here, and the resulting figure is the prefactor. The number of digits counted to the left becomes the power, or exponent, of 10. The exponent is positive if we have counted from right to left, but is *negative* if we have counted from left to right. In either case, after moving the decimal point, the remaining figure (with one digit remaining to the left of the decimal point) is the prefactor (usually with the unnecessary zeros lopped off). Try this procedure with the number 0.0000000035. In this case, we count from the original decimal point to the right nine digits, leaving the prefactor 3.5 (all the zeros to the left are dropped) and thus yielding the number in the form 3.5×10^{-9}.

The usefulness of scientific notation is easily appreciated for arithmetic with large and small numbers. Imagine that you want to calculate the total number of molecules in the Earth's atmosphere for a species that has a mixing ratio of 1 pptv. This would be written literally as 100,000,000,000,000,000,000,000,000,000,000,000,000,000 × 0.000000000001 = 100,000,000,000,000,000,000,000,000,000,000,000,000. How much easier is it to write $1 \times 10^{44} \times 1 \times 10^{-12} = 1 \times 10^{32}$. All we do is multiply the prefactors and add the exponents! Consider the examples given later, and enjoy.

Applications of Scientific Notation

To illustrate the use of scientific notation, we offer some examples. The number 550, for example, can

be written in scientific notation as 5.5×10^2. That is, $550 = 5.5 \times 10^2$. In this case, the normal decimal representation is more convenient than the scientific representation. Naturally, the simplest notation is adopted for specific numbers in the text of the book. For some numbers, the choice can be a matter of taste. Take, for example, the number of people on the Earth—approximately 5 billion. This may be written out as 5,000,000,000 or more compactly as 5×10^9. You should become familiar with both representations.

If we consider the number of molecules in Earth's atmosphere, there is little dissension. Which number is more convenient to write and which value is more immediately clear: 1×10^{44} or 100,000,000,000,000,000,000,000,000,000,000,000,000,000? In addition to compactness and immediate recognition of magnitude, numbers expressed in scientific notation are much easier to manipulate in arithmetic operations. Some examples of these operations follow.

Multiplication: The rule for multiplying numbers in scientific notation is to *multiply the prefactors* and *add the exponents* (or the powers of 10). For example,

$$1 \times 10^5 \times 1 \times 10^8 = (1 \times 1) \times 10^{8+5} = 1 \times 10^{13}$$

$$1 \times 10^{-3} \times 1 \times 10^6 = (1 \times 1) \times 10^{-3+6} = 1 \times 10^3$$

$$1 \times 10^{-2} \times 1 \times 10^{-4} = (1 \times 1) \times 10^{-2-4} = 1 \times 10^{-6}$$

$$(2,500,000,000) \times (200) = (2.5 \times 10^9) \times (2 \times 10^2) = (2.5 \times 2) \times (10^9 \times 10^2) = 5 \times 10^{9+2} = 5 \times 10^{11}$$

Division: The rule for dividing numbers in scientific notation is to *divide the prefactors*—that is, to divide the numerator prefactor by denominator prefactor—and *subtract the exponent* of the number in the denominator from the exponent of the number in the numerator. For example,

$$\frac{1 \times 10^4}{1 \times 10^3} = \frac{1}{1} \times 10^{4-3} = 1 \times 10^1 \quad \left(\text{or} \quad \frac{10,000}{1,000} = 10\right)$$

$$\frac{1 \times 10^4}{1 \times 10^5} = \frac{1}{1} \times 10^{4-5} = 1 \times 10^{-1} \quad \left(\text{or} \quad \frac{10,000}{100,000} = 0.1\right)$$

$$\frac{1 \times 10^{-6}}{1 \times 10^{-5}} = \frac{1}{1} \times 10^{(-6)-(-5)} = 1 \times 10^{-6+5} = 1 \times 10^{-1}$$

$$\frac{6 \times 10^4}{2 \times 10^3} = \frac{6}{2} \times 10^{4-3} = 3 \times 10^1 \quad \left(\text{or} \quad \frac{60,000}{2,000} = 30\right)$$

$$\frac{5 \times 10^{-3}}{2 \times 10^5} = \frac{5}{2} \times 10^{-3-5} = 2.5 \times 10^{-8}$$

Large and Small Numbers

Tables A.1 and A.2 summarize the notation and identifying names of the most common large and small numbers used in scientific discussions. These numbers may be applied to any specific quantity by attaching an appropriate suffix indicating the units of measure involved.

Using Mixing Ratios

Table A.3 lists the common mixing ratios, or mixing fractions, that are used to characterize gases in the atmosphere. The principal ratios, or fractions, used are parts per million, parts per billion, and parts per trillion (abbreviated ppm, ppb, and ppt, respectively). These mixing fractions can be expressed in parts *by volume (or number)* or parts *by mass* of air. In the former case, for example, one might specify parts per million by volume, or ppmv. For smaller fractions it would be ppbv and pptv. For mixing fractions specified as parts by mass, the corresponding abbreviations are ppmm (for parts per million by mass), ppbm, and pptm, respectively.

"Parts by volume" refers to the *fraction of the total number* of molecules in a unit volume of air of a certain kind of gas—say O_2 or CO_2 or H_2O. The *ratio* of the number of such molecules to the total number of molecules in the volume is the fraction of interest; hence the term *mixing ratio*. An

Table A.1 Large Numbers

"Normal" notation	*Scientific notation*	*Standard name*
1	1×10^{0}	One
10	1×10^{1}	Ten (*deca-*)
100	1×10^{2}	Hundred (*hecto-*)
1000	1×10^{3}	Thousand (*kilo-*)
10,000	1×10^{4}	
100,000	1×10^{5}	
1,000,000	1×10^{6}	Million (*mega-*)
10,000,000	1×10^{7}	
100,000,000	1×10^{8}	
1,000,000,000	1×10^{9}	Billion (*giga-*)
1,000,000,000,000	1×10^{12}	Trillion (*tera-*)

Table A.2 Small Numbers

"Normal" notation	*Scientific notation*	*Standard name*
0.1	1×10^{-1}	Tenth (*deci-*)
0.01	1×10^{-2}	Hundredth (*centi-*)
0.001	1×10^{-3}	Thousandth (*milli-*)
0.0001	1×10^{-4}	
0.00001	1×10^{-5}	
0.000001	1×10^{-6}	Millionth (*micro-*)
0.000000001	1×10^{-9}	Billionth (*nano-*)
0.000000000001	1×10^{-12}	Trillionth (*pico-*)

Table A.3 Mixing Fraction Definitions

One part per million	1 ppm	1×10^{-6}	one out of each million
One part per billion	1 ppb	1×10^{-9}	one out of each billion
One part per trillion	1 ppt	1×10^{-12}	one out of each trillion