Solving Absolute Value Equations

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To solve an equation involving **absolute value**, use the following property: If C is positive, then $\lvert x\rvert = C$ is equivalent to $x = \pm C$.

Special Cases for $|x| = C$:

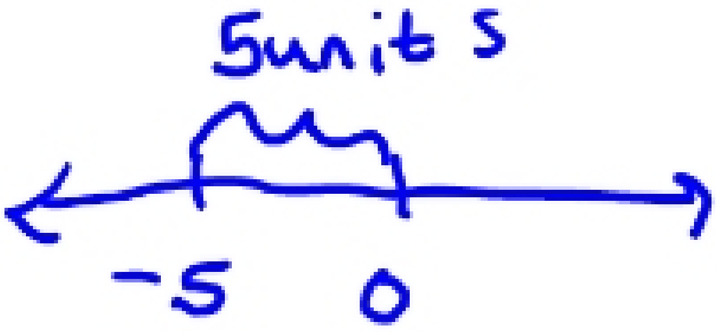
Case 1: If C is negative, then the equation $\lvert x\rvert = C$ has no solution since absolute value cannot be negative.

Case 2: The solution of the equation $\lvert x\rvert = 0$ is $x = 0$.

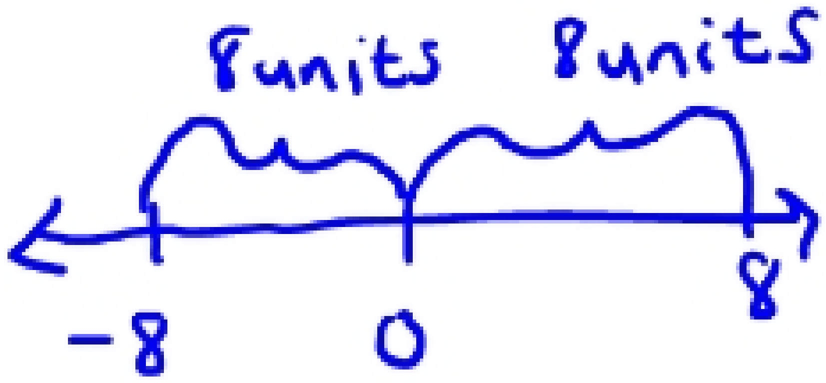
Think about absolute value as being the distance from zero.

Example 1:

$|-5| = 5$



Example 2:



$|x| = 8$

$x = \pm 8$

Example 3: Find the solution to the equation.

$|2x-4|=8$

$2x-4=8$ and $2x-4=-8$

$+4 \quad +4$ $\qquad$ $+4 \quad +4$

$\frac{2x}{2}=\frac{12}{2}$ $\qquad$ $\frac{2x}{2}=\frac{-4}{2}$

$x=6$ $\qquad$ $x=-2$

$\{-2,6\}$

Example 4: Find the solution to the equation.

$|5-2x|+6=14$ $\qquad$ $\{-3/2, 13/2\}$

$-6 \quad -6$

$|5-2x|=8$

$5-2x=8$ and $5-2x=-8$

$-5 \quad -5$ $\qquad$ $-5 \quad -5$

$\frac{-2x}{-2}=\frac{3}{-2} \rightarrow x=-3/2$ $\qquad$ $\frac{-2x}{-2}=\frac{-13}{-2}$

$x=13/2$

Example 5: Find the solution to the equation.

$20+|2x+4|=15$

$-20 \quad -20$

$|2x+4|=-5$

No Solution

Absolute value can not equal a negative number

Example 6: Find the solution to the equation.

$$\frac{1}{4}|2x-6|=4$$

$\left(\frac{4}{1}\right)\left(\frac{1}{4}\right)|2x-6|=\frac{4}{1}(4)$

$|2x-6|=16$

$2x-6=16$ and $2x-6=-16$

$+6 \quad +6$ $\qquad$ $+6 \quad +6$

$\frac{2x}{2}=\frac{22}{2}$ $\qquad$ $\frac{2x}{2}=\frac{-10}{2}$

$x=11$ $\qquad$ $x=-5$

$\{-5, 11\}$

Solving Absolute Value Inequalities

Special Cases:

Case 1:

* is always positive

If C is negative, then:

a) The inequality $|x|<C$ has no solution.

b) The inequality $|x|\le C$ has no solution.

positive < negative

false

c) Every real number satisfies the inequality $|x|>C$.

d) Every real number satisfies the inequality $|x|\ge C$.

positive > negative

true

Case 2:

* always positive

a) The inequality $|x|<0$ has no solution.

$0<0$ false

b) The solution of the inequality $|x|\le 0$ is $x=0$.

$0\le 0$ true

c) The inequality $|x|>0$ in equivalent form is $x<0$ or $x>0$.

d) Every real number satisfies the inequality $|x|\ge 0$.

true