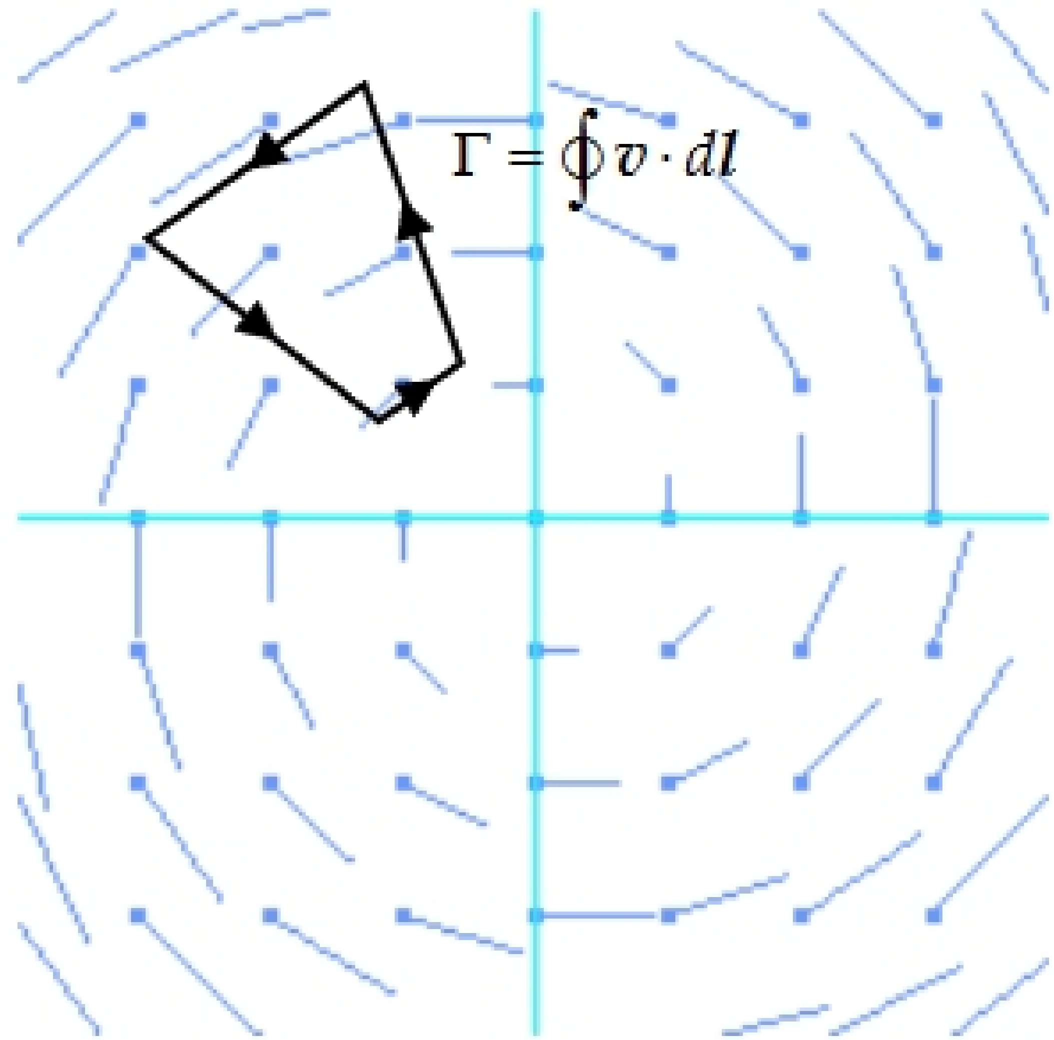
Today in Physics 217: vector integrals

Three useful generalizations of the fundamental theorem of calculus:

- ❑ Gradient theorem
- ❑ Gauss' divergence theorem
- ❑ Stokes' curl theorem



Integral vector calculus

Fundamental theorem of calculus for a function of one variable:

$$\int_a^b \frac{df(x)}{dx}dx = f(b) - f(a)$$

In vector calculus, there are three different kinds of derivatives – gradient, divergence and curl – so there are three different analogues of the fundamental theorem of calculus:

- the **gradient theorem**:

$$\int_C \nabla T \cdot d\boldsymbol{l} = T(\boldsymbol{b}) - T(\boldsymbol{a})$$

  where the integral is taken along the curve C, and $\boldsymbol{a}$ and $\boldsymbol{b}$ are the position vectors of the endpoints of C.

Integral vector calculus (continued)

- **Stokes' theorem**, for curls:

$$\int_S (\nabla \times \boldsymbol{v})\, d\boldsymbol{a} = \oint_C \boldsymbol{v} \cdot d\boldsymbol{l}$$

  where the integral on the left is carried out over a surface *S*, and that on the right is carried out all the way around the curve *C* that bounds *S*.

- And (Gauss') **divergence theorem**:

$$\int_V (\nabla \cdot \boldsymbol{v})\, d\tau = \oint_S \boldsymbol{v} \cdot d\boldsymbol{a}$$

  where the integral on the left is carried out over a volume *V*, and that on the right over the surface *S* that bounds *V*.

Illustrating these theorems one by one…