# EE468G NOTES (1)

Reading assignment: Vectors

Contents: **Unit Vectors, Vectors, Vector operations**
**Vectors in three coordinate systems**

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**Vector**: A quantity that has magnitude and direction

Examples: Force, velocity, field intensity

Notation and writting: **A** or $\bar{\mathrm{A}}$ or $\vec{\mathrm{A}}$

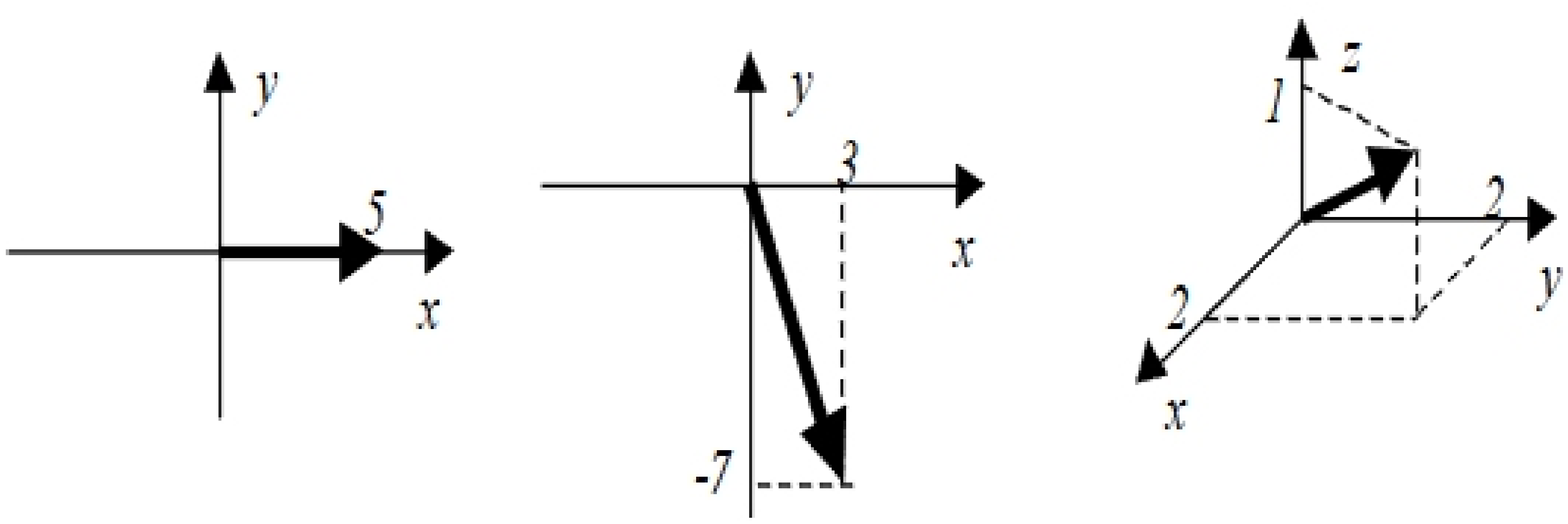
Unit vector: a vector whose magnitude is 1. Examples:

$\hat{a}_x = \hat{x}$ is a unit vector pointing in x-direction,

$\hat{y}$ is a unit vector pointing in y-direction.

Vector representation in rectangular coordinate system: component format

$$\vec{A} = 5\hat{x} \qquad \vec{B} = 3\hat{x} - 7\hat{y} \qquad \vec{C} = 2\hat{x} + 2\hat{y} + \hat{z}$$




In general, a vector has three components $A_x$, $A_y$, $A_z$:

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\ ,$$

Component $A_x$ is the projection of vector $\vec{A}$ on x-axis.

**Calculation of magnitude**: $\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$ . Examples:

$$\vec{B} = 3\hat{x} - 7\hat{y}, \qquad \left|\vec{B}\right| = \sqrt{3^2 + (-7)^2 + 0^2} = \sqrt{58}$$

$$\vec{C} = 2\hat{x} + 2\hat{y} + \hat{z}, \qquad \left|\vec{C}\right| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

**Direction of a vector**: The direction of a vector can be described by three angles that the vector made with the x-axis, y-axis, and z-axis, respectively. In the above examples,

$$\vec{B} = 3\hat{x} - 7\hat{y}, \qquad \left|\vec{B}\right| = \sqrt{3^2 + (-7)^2 + 0^2} = \sqrt{58}$$

It makes an angle of $\cos^{-1}\left(3/\sqrt{58}\right) = 66.8^o$ with x-axis,

It makes an angle of $\cos^{-1}\left(-7/\sqrt{58}\right) = 156.8^o$ with y-axis,

It makes an angle of $\cos^{-1}\left(0/\sqrt{58}\right) = 90^o$ with z-axis.

In general, if $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$, the angles it makes with x-, y-, and z-components are: $\alpha_x, \alpha_y$, and $\alpha_z$, where

$$\alpha_x = \cos^{-1}\left(A_x/\left|\vec{A}\right|\right), \quad \alpha_y = \cos^{-1}\left(A_y/\left|\vec{A}\right|\right), \quad \alpha_z = \cos^{-1}\left(A_z/\left|\vec{A}\right|\right)$$

Example: Let $\vec{A} = 6\hat{x} + 2.5\hat{y} - 0.8\hat{z}$, calculate $\left|\vec{A}\right|$, $\alpha_x, \alpha_y$, and $\alpha_z$.

Solution:

$$\left|\vec{A}\right| = \sqrt{6^2 + 2.5^2 + (-0.8)^2} = \sqrt{42.89} = 6.55$$

$$\alpha_x = \cos^{-1}(6/6.55) = 23.65^o$$

$$\alpha_y = \cos^{-1}(2.5/6.55) = 67.56^o$$

$$\alpha_z = \cos^{-1}(-0.8/6.55) = 97.02^o$$

**Vector operations**: Addition and subtraction

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\,, \qquad \vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

Addition of $\vec{A}$ and $\vec{B}$: $\vec{A}+\vec{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$

Subtraction of $\vec{A}$ and $\vec{B}$: $\vec{A}-\vec{B} = (A_x - B_x)\hat{x} + (A_y - B_y)\hat{y} + (A_z - B_z)\hat{z}$

The results of addition and subtraction are vectors.
The addition and subtraction operation satisfy the commutative law and distributive law.

Example: Let $\vec{A} = 6\hat{x} + 2.5\hat{y} - 0.8\hat{z}$, $\vec{B} = -3\hat{x} + 2\hat{y} + 6\hat{z}$ calculate $\vec{A}+\vec{B}$, and $\vec{A}-\vec{B}$

Solution:

$$\vec{A}+\vec{B} = (6-3)\hat{x} + (2.5+2)\hat{y} + (6-0.8)\hat{z} = 3\hat{x} + 4.5\hat{y} + 5.2\hat{z}$$

$$\vec{A}-\vec{B} = (6+3)\hat{x} + (2.5-2)\hat{y} - (6+0.8)\hat{z} = 9\hat{x} + 0.5\hat{y} - 6.8\hat{z}$$

Graphical representation of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$