Exam 4 – v1

Physics 2760

FS 2013

Lab Section

Last Name_____

First Name_____

ID # _____

Solutions

This is a closed book exam. I understand, pursuant to University Regulations on academic honesty, that I am not to use any notes or information other than what is in the official, non-annotated formula sheet.

Signature_____

→ For multiple choice questions, please make sure that you **circle the letter for the answer which you believe to be correct and only that answer**. If more than one answer is circled for the same problem, you will not receive credit for it.

→ **For full credit show your work for solutions to questions that require calculations**. Write the equation from where you start to solve the problem and show your math flowing from it for full credit. **No shown work, no credit!**

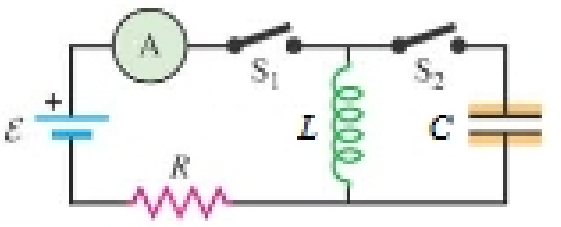
→ Surround your final answer with a box and **make sure you include units for your final answer**, otherwise you will be penalized!

→ Don't get hung up on questions. If you find yourself spending too much time on a question, skip it and come back to it later. **Relax, read carefully, think – and then read everything again.**

→ During the exam, if you have questions please **raise your hand** and the instructor will come to you and provide help.

→ **The last page is the formula sheet.** Feel free to tear it off. You may keep the formula sheet after the exam.

1. In the circuit shown the emf is 40 V, R = 10 Ω, L = 2 mH and C = 6 μF. While switch S_2 is open, switch S_1 is closed.



a) What is the voltage across the inductor at the instant switch S_1 is closed? (1 point)

$V_L = \mathcal{E} = 40V$

b) What is the current in the circuit after a long time? (1 point)

$I = \frac{\mathcal{E}}{R} = \frac{40}{10} \Rightarrow \boxed{I = 4\ A}$

c) At what time is the current in the circuit half of its maximum value? (3 points)

$i = I_{max}\left(1 - e^{-t/\tau}\right) \Rightarrow \frac{1}{2} = 1 - e^{-t/\tau}$ $\quad e^{-t/\tau} = \frac{1}{2}$

$i = \frac{1}{2} I_{max}$

$-\frac{t}{\tau} = \ln\frac{1}{2} \Rightarrow t = \tau \cdot \ln 2 = \frac{L}{R}\ln 2 \Rightarrow \boxed{t = 0.138\ ms}$

d) S_1 is opened and S_2 is closed. Find the maximum charge on the capacitor in the LC circuit. (4 points)

$\frac{1}{2}LI^2 = \frac{1}{2}\frac{Q^2}{C} \Rightarrow Q^2 = LI^2 \cdot C \quad Q = I\sqrt{LC}$

$Q = 4\sqrt{(2\times10^{-3})(6\times10^{-6})} \Rightarrow \boxed{Q = 4.38\times10^{-4}\ C}$

e) What is the current in the inductor when the capacitor has maximum charge? (1 point)

$I_L = 0$ when $Q = Q_{max}$

f) At what time is the energy stored in the inductor equal to the energy stored in the capacitor? (4 points)

$\frac{1}{2}Li^2 = \frac{1}{2}\frac{q^2}{C}$ where $\begin{cases} i = I\sin\omega t = \omega Q \cdot \sin\omega t \\ q = Q\cos\omega t \end{cases}$ $\quad \omega = \frac{1}{\sqrt{LC}}$

$L(\omega Q)^2\sin^2\omega t = \frac{Q^2}{C}\cos^2\omega t \Rightarrow \frac{\sin^2\omega t}{\cos^2\omega t} = \frac{Q^2}{\omega^2 Q^2 C L} = 1$

$\Rightarrow \tan^2\omega t = 1 \Rightarrow \omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega} \Rightarrow \boxed{t = 0.085\ ms}$

2. An L–R–C series circuit has R= 290 Ω. At the frequency of the source, the inductor has reactance X_L= 900 Ω and the capacitor has reactance X_C= 500 Ω. The maximum voltage across the inductor is 440 V.

a) What is the maximum current in the circuit? (2 points)

$V_L = I \cdot X_L \Rightarrow I = \frac{V_L}{X_L} = \frac{440}{900} \Rightarrow \boxed{I = 0.48\ A}$

b) What is the maximum voltage across the resistor? (2 points)

$V_R = I \cdot R \Rightarrow V_R = (0.48)(290) \Rightarrow \boxed{V_R = 139.2\ V}$

c) What is the phase angle? (2 points)

$\tan\varphi = \frac{X_L - X_C}{R} = \frac{900 - 500}{290} = 1.38 \Rightarrow \boxed{\varphi = 54^\circ}$

d) What is the maximum voltage of the source? (2 points)

$V = I \cdot Z$ where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{(290)^2 + (900-500)^2} \cong 494\ \Omega$

$V = (0.48)(494) \Rightarrow \boxed{V = 237.12\ V}$

e) What is the average power delivered by the source? (3 points)

$P_{avg} = \frac{1}{2} I V \cos\varphi = \frac{1}{2}(0.48)(237.12)\cos 54^\circ \Rightarrow \boxed{P_{avg} = 33.45\ W}$

f) Draw a phasor diagram of the voltages and current in this circuit and make sure you indicate all angles in your drawing. (3 points)