Modern Physics (PHY 3305) Lecture Notes

Waves and Uncertainty (Ch. 4.4-4.5, 5.1-5.2)

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Review

tags: lecture

- We moved to a description of nature that focuses on the wave nature of matter
  - We know that the *relevant dimensions* of a specific problem will lead to more wave-like or particle-like results
- The wave nature of matter is described by the Shroedinger Wave Equation (SWE) that, like Newton's Laws or Conservation of Energy, is motivated by experimental results and is not derivable from first principles.
- We discussed complex numbers and functions, in preparation for handling the complex wave function allowed by the SWE.
- We discussed the meaning of the wave function - that it represents PROBABILITY PER UNIT LENGTH (or VOLUME in 3-dimensions), referred to as PROBABILITY DENSITY.

Let us begin with a question about atoms

QUESTION: do atoms radiate energy all the time?

DISCUSSION: why not? After all, classically we think of the electron as going around the atom like a planet orbiting a sun. If its orbiting, its moving and being bent in that orbit by the Coulomb force. There is a place where the charge is, and where it is not - the charge density is changing and thus the electron should be radiating energy like crazy! That's what EM predicts for such motion.

DISCUSSION: what did you learn about the atom in your homework? Can it just be treated classically?

Answer: no. Today, we'll begin to see what happens when we think of the electron not as being in classical orbit around the nucleus, but being BOUND to the nucleus.

We'll begin first by picking up our discussion where we left off and then talking about uncertainty. Then we'll jump into what the SWE teaches us about BOUND STATES.

The Schroedinger Wave Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

QUESTION: what, exactly, is this equation comparing for the wave function?

DISCUSSION:

- You have a second-derivative with respect to space. This is telling us about the CURVATURE of the wave function in space.
- You have a first-derivative of the wave function with respect to time. This is telling us about the change of the wave function in time.
- So the SWE relates spatial curvature to temporal variation.

Solutions to the Schroedinger Equation: Plane Waves

The whole game of using the Schroedinger equation is the same as using Newton's laws of motion, or relativity:

- First, identify the players in the system
- Second, identify constraints on the systems - are there boundaries to the problem, are there forces, etc?
- Third, convert the above into math and identify a solution to the equation that satisfies all known constraints
- Fourth, with the solution in hand, compute amplitudes-squared to solve for the measurables in the problem

The first solution we will explore is the wave function describing free particles - particles in constant motion free of external forces. The wave function that describes this situation is:

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

where A is a constant. If we insert this into the Schroedinger Wave Equation, we obtain the following equation:

$$\frac{\hbar^2 k^2}{2m} = \hbar\omega$$

The functional dependence on space and time cancels out, and we conclude that this function obeys Schroedinger's equation for all space and time, provided that k and ω are related as above.

So how do plane waves relate to particles? What happens if we insert:

$$p = \hbar k$$

and

$$E = \hbar\omega$$

We find:

$$\frac{p^2}{2m} = E$$

Well, if we are describing a classical (low-velocity) particle in motion, $p = m\nu$ and we find that:

$$\frac{1}{2}m\nu^2 = E$$

Which is just an expression that the energy of a particle whose velocity is much smaller than that of light has energy equal to its kinetic energy. This we already know from classical mechanics.