Modern Physics (PHY 3305) Lecture Notes

Describing Nature as Waves (Ch. 4.3-4.4)

SteveSekula, 10 February 2010 (created 13 December 2009)

Review

tags: lecture

- We concluded our discussion of light/radiation as particles
- We tried to answer the question: is light a wave or a particle?
  - We discussed duality, the complementarity of the wave-like and particle-like aspected of light
- We asked "why is light special?" and confronted de Broglie's hypothesis: matter has wave and particle behavior, too!
- We looked for evidence of the wave nature of matter (atoms, Bragg scattering)
- We wrote down the wave properties of matter in terms of the particle properties
- We thought about why the wave nature of matter is so undetectable on terrestrial scales

Matter Waves: Redux

The de Broglie hypothesis for the wave nature of matter was to simply re-write the equations describing light's dual nature:

$$E_{light} = hf$$

$$p_{light} = h/\lambda$$

and produce the predicted equations for matter:

$$f_{matter} = h/E$$

$$\lambda_{matter} = h/p$$

We also found another connection between the wave and particle aspects of light and matter - that of PROBABILITY. We observed that for light, the probability of finding a photon in some location is proportional to the intensity of the wave-aspect of the light, which is in turn proportional to the square of the amplitude of the wave. Thus

$$P_{photon} \propto A^2_{\mathrm{EMwave}}.$$

This was also observed for electrons (and other matter): the probability of finding one in a given location appeared to be connected to an underlying intensity of some wave, so again

$$P_{matter} \propto A^2_{\mathrm{matterwave}}$$

Matter at Rest: Infinite Wavelength?

An outstanding question from last time was:

- Doesn't the wavelength equation for matter suggest that when matter is absolutely still (at rest), its wave nature should be apparent to all because its wavelength is infinite?

This is a very insightful question, and in fact the answer was given in class during the class discussion: is anything ever truly "at rest" - that is, $p = 0$ exactly? This question is going to become very important in the next few lectures. How do we "know" that something is truly at rest, and can something ever truly be exactly at zero motion?

We'll return to this question very soon, but for now consider the fact that I as a person am never truly at rest: my nerves twitch, my cells divide - I am fundamentally in constant motion. So the wave-nature of my matter is held in check by the fact that it's constantly in motion.

Matter in Motion: Changing the Wave Properties

Another consequence of these equations lies in the answer to the following question:

- What happens to the wave properties of matter when you increase its speed?

The answer is: the wavelength becomes shorter as the speed increases. Shorter wavelengths mean you can probe smaller and smaller structures in nature, by looking at how matter waves scatter off of targets.

- What are some examples of using matter to probe short distance scales?
  - electron microscope
  - particle accelerators

- If you have to probe small structures, is it better to use light or matter? Why?
  - DISCUSSION

Some ingredients in the discussion:

- visible light microscopes are limits to 100-200 nm resolution due to diffraction in the microscope elements and aberrations in optics.
- you could instead scatter x-rays off of targets
- To obtain an image of the same target (e.g., a 1nm resolution of a 400nm structure), you need light of 1nm or matter of wavelength 1nm.
- Compare the probe particles:
  - Momentum is the same for both: $p = h/\lambda \approx 6.6 \times 10^{-25} J \cdot s/m$
  - The energy of the light (x-rays, given the wavelength) is $E = hf = hc/p \approx 1200 keV\ (2 \times 10^{-16} J)$
    - Light is "hard" to work with because you need good optics to focus the light.
  - The speed of electrons with this momentum: $u \approx p/m = 7.3 \times 10^5 m/s \approx 0.002c$
    - Is that "cheap"? What kind of voltage would you need to apply to get electrons up to that speed? ANSWER: About 1.5V. THAT'S PRETTY CHEAP! Just need a source of electrons (e.g. a hot filament).
    - To focus the electrons just needs magnetic fields, which can be controlled very precisely and in real time (doing this with optics is much harder)

Matter waves are quite useful, since the inherent wavelength of a massive object is already quite small, and charged matter can be controlled