

1. The velocity of an object traveling in a straight line is given by $v(t) = 20 - \sqrt{t}$ ft/sec.

Find the average acceleration (average rate of change of velocity) of the object from

a) $t=1$ to $t=4$ sec. b) from $t=4$ to $t=7$ sec.

Use your calculator to find $\lim_{h \rightarrow 0} \frac{v(4+h) - v(4)}{h}$.

2. Use the calculator to find the limits.

$$a) \lim_{x \rightarrow 0} \frac{e^x - 1}{x} \quad b) \lim_{h \rightarrow 0} \frac{(2+h)^5 - 32}{h} \quad c) \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$$

$$3. f(x) = \begin{cases} 4 \sin x & x < 0 \\ \ln(x+1) & 0 \leq x < e-1 \\ \frac{1}{x} & e-1 \leq x \end{cases}$$

Find each limit or state DNE for a) $c=0$ and b) $c=e-1$

$$\lim_{x \rightarrow c^-} f(x), \quad \lim_{x \rightarrow c^+} f(x), \quad \lim_{x \rightarrow c} f(x)$$

4. $f(x) = \frac{x^2 - 9}{x^2 - x - 6}$ Find the limits $\lim_{x \rightarrow c^-} f(x)$, $\lim_{x \rightarrow c^+} f(x)$, $\lim_{x \rightarrow c} f(x)$. State DNE for any that do not exist.

a) $c=3$

b) $c=-2$

5. Evaluate each limit.

$$a) \lim_{x \rightarrow 0} \ln|x| \quad b) \lim_{x \rightarrow 0} x \ln|x| \quad c) \lim_{x \rightarrow \frac{\pi}{2}} e^{\sin x}$$

6. Evaluate each limit.

$$a) \lim_{x \rightarrow \infty} \frac{8x^3 - 5x^2 + 65}{x^2 + 9} \quad b) \lim_{x \rightarrow -\infty} \frac{8x^3 - 5x^2 + 65}{x^2 + 9}$$

$$c) \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 9x} - x \right) \quad d) \lim_{x \rightarrow \infty} \frac{5e^x + 7}{4e^x - 2}$$

$$e) \lim_{x \rightarrow -\infty} \frac{5e^x + 7}{4e^x - 2}$$

7. At what x -values if any is f not continuous? Give a graphical reason and a definition of continuity reason for each discontinuity.

$$a) f(x) = \begin{cases} \frac{x^2 - 9}{x^2 - x - 6} & x \neq 3, \quad x \neq -2 \\ 1 & x = 3, \quad x = -2 \end{cases}$$

$$b) f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$c) f(x) = \begin{cases} x \tan\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$8. f(x) = \begin{cases} x^2 & x < 1 \\ 3x + b & 1 \leq x \end{cases}$$

a) Find the value of b so that f is continuous at $x = 1$.

b) For the value of b found in part a, find the average rate of change of $f(x)$ from $x=0$ to $x=1$.

c) For the value of b found in part a, find the average rate of change of $f(x)$ from $x=1$ to $x=2$.

d) Guess $\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h}$ and $\lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h}$. What do you see in the graph at $x=1$?

9. Find the average rate of change of $f(x) = \sqrt[3]{x}$ from $x=0$ to $x=1$. Does $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$ exist?