

## Algorithms for Adding and Subtracting Whole Numbers

An **algorithm** is a step by step procedure for solving a problem. In this section we will discuss several algorithms for adding and subtracting whole numbers. Here, we will learn about why you sometimes “carry” in addition and “borrow” in subtraction.

**Example 1:** Consider the problem

$$\begin{array}{r} 1 \\ 28 \\ +45 \\ \hline 73 \end{array}$$

One way to think about this problem is as follows: 28 is 2 tens and 8 ones, 45 is 4 tens and 5 ones. To add 45 and 28, add the “ones” first to get 13, which is 1 ten and 3 ones. Then add up *all* the tens. We have 2 tens from the 28, 4 tens from the 45, and 1 ten from the ones column. So we have a total of 7 tens and 3 ones and so our answer is 73. This is best seen using manipulatives like blocks or strips/mats.

The process where we “carried” the one is called an **exchange**. The idea is that we exchanged 10 ones for 1 ten.

**Example 2:** In the strip model, make exchanges to represent 15 squares, 11 strips and 2 mats with the smallest number of pieces.

### The Addition Algorithm:

- With units, squares, and mats add 135 and 217.
  1. Represent both numbers using mats for hundreds, strips for tens, and squares for ones.
  2. Combine all the manipulatives and make exchanges so that you use the least number of pieces.
  3. Count all the pieces. The answer is 352.
- Use the **place value card** approach to add 135 and 217
  1. Mark off three cards with squares for ones, tens, hundreds, and so on.
  2. On one card record the number of ones, tens, and hundreds for the first addend. Do the same for the second addend.
  3. On the third card mark off the total number of ones, tens, hundreds, and so on from the first two cards. Make exchanges so that there are no more than 9 ones, tens, hundreds, and so on.
  4. Read the answer of the third card.
- With **place value diagrams, instructional diagrams, and the final algorithm** add 135 and 217. Record the following

1. Place value diagram

100s	10s	1s
1	3	5
2	1	7
3	4	(12)
3	5	2

2. Instructional algorithm

$$\begin{array}{r}
 135 \\
 +217 \\
 \hline
 12 \\
 40 \\
 300 \\
 \hline
 352
 \end{array}$$

3. Final algorithm

$$\begin{array}{r}
 1 \\
 135 \\
 +217 \\
 \hline
 352
 \end{array}$$

- As a note, teaching students the word “carrying” is somewhat misleading. What is really happening when we do addition is a sequence of exchanges. You might consider teaching your students to call this procedure *exchanging*, *trading*, or *regrouping* instead.

**Example 3:**

Consider the problem

$$\begin{array}{r}
 2 \\
 32 \\
 -28 \\
 \hline
 4
 \end{array}$$

We can think about this problem as taking 2 tens and 8 ones away from 3 tens and 2 ones. Since we can’t take 8 ones away from the 2 ones in 32, we take one of the tens and “break it” into ones. So we have rewritten 32 as 2 tens and 12 ones. Now we take 8 ones away from the twelve to get 4 ones and two tens from two tens to get 0 tens.

**The Subtraction Algorithm:**

- Subtract 154 from 322 using mats, strips, and units.
  1. Represent both the minuend and subtrahend using mats for hundreds, strips for tens, and squares for ones.
  2. Start with the units. In this case we can’t subtract 2 from 4, so we must exchange one strip from 322 for 10 units. Then we get 12 ones from which we take 4 to get 8 ones in total.
  3. We traded in one of the strips in 322, so there is only one strip left for this number. We can’t subtract 5 tens from 1 tens so we trade one of the mats for ten strips. Then there are 11 strips left. From these we subtract 5 strips to get 6 strips total.

