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Sets, Counting and The Whole Numbers

Question? Why do we have numbers? How do you think they were invented? What different ways are numbers used?

Numbers. Three types:

- **nominal** or **identification**: A number used as a label. Ex.: Phone numbers, student ID numbers.
- An **ordinal** number is a number from a list of ordered numbers. Ex.: raffle ticket numbers, year 2009.
- A **cardinal number** measures the number of objects in a set. Ex.: Fifteen lottery tickets. 32 books on the shelf.

One-to-one correspondence of sets and equivalent sets.

- Recall, two sets are **equal** if they contain exactly the same elements. Ex. If $A = \{x, y, z\}$ and $B = \{z, y, x\}$ then A equals B and we write $A = B$.
- We say that there is a **one-to-one** correspondence between the sets A and B if every element of the set A can be paired off with exactly one element of B so that every element of A has exactly one partner from B and every element of B has exactly one partner from A.
- More mathematically, we say *there is a one-to-one correspondence between the sets A and B if there is a way to assign exactly one element of B to every element of A which uses all the elements of B exactly one time.*
- When there is a one-to-one correspondence between the sets A and B we say the sets are **equivalent**. We write $A \sim B$.
- Examples.: $C =$ { Barack Obama, Joe Biden, Hillary Clinton } and $P =$ { president, vice-president, secretary of state }. $A = \{a, b, c, d\}$ and $N_4 = \{1, 2, 3, 4\}$.

- Note that the examples show that $A \sim P$ is *not the same* as $A = B$. So being equivalent is not the same as being equal. However if $A = B$ then certainly $A \sim B$.

The whole numbers and the size of a set.

- Recall that

$$N = \{1, 2, 3, 4, ...\}$$

  is the set of natural or counting numbers. How many elements are in this set?

- We say a set is **finite** if it is equivalent to either $\emptyset$ or to the set $\{1, 2, 3, 4, ..., n\}$ where n is a natural number.

- If a set is not finite it is called **infinite**. Ex.: Odd numbers, prime numbers.

- We will write $n(A)$ to denote the number of elements in the set A. The number of elements in a set is called the **cardinality** or **cardinal number** of the set.

- The set W is the set set of **whole numbers**. This is the set of numbers which could be the cardinal number of a set. So of course

$$W = \{0, 1, 2, 3, ...\}.$$

- Examples.: Find (a) the cardinality of the set of days in the week, (b) $n(E)$ where E is the set of even numbers (c) $n(\emptyset \cap W)$ (d) cardinal number of $\{0\}$ and (e) $n(A)$ where $A = \{x \in W | x^2 < 100\}$.

Models of Whole Numbers. Tiles, blocks, strips, number line where whole numbers represent the distance from 0. Page 95.

Ordering the whole numbers.

- Recall the definition of a subset: $A \subset B$ if every element of A is also an element of B.

- We will say A is **a proper subset** of B if $A \subset B$ and $A \neq B$.

- Now let $a = n(A)$ and $b = N(B)$ be whole numbers where A and B are finite sets. If A is equivalent to a proper subset of B then **a is less than b** and we can write $a < b$.

- Ex.: Demonstrate that $2 < 5$ with models: objects, tiles, strips, number lines.

More problems with Venn-diagrams. Use Venn diagrams to solve the following problems:

- There are 150 senators on the student senate and there are three optional committees: Greek life (G), academic affairs (A), and social life (S). There are 30 senators on G, 50 on A, and 40 on S. Also, 10 senators are on $A \cap G$, 10 are on $A \cap S$, 15 are on $S \cap G$. There are 5 senators on all three committees. How many senators did not volunteer for any committees?

- There are 25 people at the ice cream social. 15 have chocolate; 15 have vanilla, 10 have strawberry; 10 have both chocolate and vanilla; 5 have both chocolate and strawberry; 5 have both strawberry and vanilla; 3 people have all three. How many people have no ice-cream? Answer:25-23=2