Distributed Topology Control in Wireless Sensor Networks with Asymmetric Links

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Abstract— **Topology control with per-node transmission power adjustment in wireless sensor networks has been shown to be effective with respect to prolonging network lifetime via power conservation and increasing network capacity via better spatial bandwidth reuse. In this work, we consider the problem of topology control in a network of heterogeneous wireless devices with different maximum transmission ranges, where asymmetric wireless links are not uncommon. In such an environment, we present a distributed topology control algorithm to calculate the per-node minimum transmission power, so that (1) reachability between any two nodes is guaranteed to be the same as in the initial topology; and (2) nodal transmission power is minimized to cover the least number of surrounding nodes. Analysis and simulation results demonstrate the correctness and effectiveness of our proposed algorithm.**

I. Introduction

Wireless sensor networks are formed by a collection of *power-conscious* wireless-capable sensors without the support of pre-existing infrastructure, possibly by unplanned deployment. Topology control via per-node transmission power adjustment has been shown to be effective in extending network lifetime and increasing network capacity (due to better spatial reuse of spectrum). The flip side of the coin is, with a reduced transmission range on each node, basic *reachability* from one node to another may be jeopardized. This problem is further exacerbated when we consider a network of heterogeneous wireless devices with *different maximum transmission ranges*, where **asymmetric** (or *uni-directional*) wireless links are not uncommon in the topology.

There exists considerable previous work addressing the topology control problem of minimizing nodal transmission power, with guarantees of network connectivity. For example, Wattenhofer *et al.* [1] proposed a fully distributed algorithm that only relies on directional information between nodes. Ramanathan *et al.* [2] presented a centralized topology control algorithm, along with a distributed heuristic. It has not discussed, however, guarantees on connectivity. Unlike the above deterministic guarantee of connectivity, Santi *et al.* [3] analyzed the connectivity of a sensor ad hoc network using a probabilistic approach in order to find out the minimum transmission power to be used at all nodes. The lower and upper bound on the probability of network connectivity are derived for certain transmission range assignments. Lloyd *et al.* [4] continued research towards this direction, with sound theoretical analysis on the properties of generic topology control protocols in minimizing the maximum power adopted and the total energy consumed in the network. Rodoplu *et al.* [5] presented an topology control algorithm that is most similar to our proposal, requiring location information and working on vicinity topologies on each node in a distributed fashion. With the wealth of results related to topology control, *none* of the previous work has extensive discussions on the problem introduced by asymmetric (uni-directional) wireless links, and proposed algorithms tailored to this specific scenario.

When the existence of asymmetric links is not assumed in order to simplify the problem to tractable theoretical models, the following two issues are unavoidably introduced. First, if all links in the original topology are symmetric, it is impossible to assume the use of different transmission ranges among nodes. Second, if asymmetric links are allowed to exist in the finalized topology, the derived minimum-power topology may become more power-efficient since transmission ranges may be further reduced.

By placing asymmetric wireless links in the scope and spotlight of our work, we design a distributed topology control algorithm that enjoys the following favorable properties: First, the algorithm converges rapidly. For stationary sensor networks, the minimum power topology is finalized in a single pass. Second, the algorithm is not complex computationally, while still effective to guarantee the bi-directional multi-hop reachability between nodes in the network. Third, since information exchange between nodes is limited to the local neighborhood, the algorithm scales well to large networks.

The remainder of the paper is organized as follows. Sec. II describes our system model. Our distributed topology control algorithm is presented and analyzed in Sec. III and IV. In Sec. V, we show the correctness and effectiveness of our algorithm with simulation results. We conclude the paper and summarize its highlights in Sec. VI.

II. Model

In this work, we consider a wireless sensor network as a network of heterogeneous sensors, referred to as *nodes*. All nodes are arbitrarily deployed in a two-dimensional plane. Each node is equipped with an omni-directional antenna with adjustable transmission power. Since nodes are heterogeneous, they have different maximum transmission powers and radio ranges. For node i, we use P_i to denote its transmission power, P_i^{max} as its maximum transmission power (or, alternatively, *full power*), and P_{ij} as the transmission power required for node i to reach node j. Under the assumption that the transmission medium is symmetric (and that asymmetric links are only due to the different ranges), we have $P_{ij} = P_{ji}$. Since $P_i^{\max} \neq P_j^{\max}$ for $i \neq j$, in the situation where $P_i^{\max} \geq P_{ij} > P_j^{\max}$, there exists an asymmetric link $\overrightarrow{L_{ij}}$ in the network topology since $P_{ji} > P_j^{\max}$ (impossible for j to reach i with its full power). Our work focuses on such asymmetric links.

Due to the existence of asymmetric links, the topology where each node transmits with its maximum transmission power is naturally a directed graph, referred to as the *maximum topology* $\overrightarrow{G} = (V, \overrightarrow{L})$. $\overrightarrow{G}$ can be either strongly connected, weakly connected, or disconnected. In a *strongly connected* $\overrightarrow{G}$, there is a directed, possibly multi-hop, path from any source to any destination. In a *weakly connected* $\overrightarrow{G}$, there exists pairs of nodes that only one of them can reach the other via multiple hops. Finally, in a *disconnected* $\overrightarrow{G}$, their exist pairs of nodes that can not reach each other.

The objective of our distributed topology control algorithm is to derive a *minimum-power topology* $\overrightarrow{G'}$ that is strongly connected, guaranteeing multi-hop reachability from any source to any destination in the directed graph. We assume that the algorithm begins with a strongly connected maximum topology. The topology control algorithm is assumed to start with a *strongly connected* $\overrightarrow{G}$. The need of such an assumption may be easily derived by contradiction.

When a node, such as node i, sends a message in the network, it broadcasts the message at a specific power level, in the range of $(0, P_i^{\max}]$. We use the path loss model commonly adopted by previous work [5], [6], where the power of the received signal is found to have a distance dependence of $1/d^n$, where d is the propagation distance and exponent n ranges from 2 to 5 depending on the environment. Despite this simplifying assumption, our algorithm works correctly with any path loss models as long as a node knows the path loss models of its neighboring nodes, which is achievable via local notifications without violating the distributed nature of the algorithm. We further assume that location information (x_i, y_i) is available to node i, for all nodes in the network. However, node i is not aware of the locations of other nodes in the network.

Finally, our solution is based on the existence of a MAC layer protocol [7] or sub-routing layer [8] that are aware of asymmetric links to ensure the network protocols function correctly with the presence of asymmetric wireless links.

III. Algorithm

Our topology control algorithm starts with the strongly connected maximum topology $\overrightarrow{G}$ of a wireless sensor network, and generates its minimum-power topology $\overrightarrow{G'}$ with a guarantee of the same bi-directional (and possibly multi-hop) reachability between any node-pairs. It is a fully distributed algorithm since each node runs the algorithm based on its local information, and possibly at different times. No synchronization is required among nodes in the network. After every node in $\overrightarrow{G}$ finishes running the algorithm, the network topology converges to $\overrightarrow{G'}$. Without loss of generality, we focus on an arbitrary node, i, and present the algorithm in three phases.

A. Phase 1: Establishing the vicinity topology

The skeleton is described as follows. Node i broadcasts a message, referred to as the *initialization request* (IRQ) message, using its maximum transmission power $P_i^{\max}$. The set of nodes that receive the IRQ message are referred to as the *vicinity nodes* of node i, denoted as V_i. The IRQ message includes the location of $i, (x_i, y_i)$, as well as $P_i^{\max}$. Upon receiving such an IRQ message, each node j in V_i replies to node i with an *initialization reply* (IRP) message, with its location (x_j, y_j) and $P_j^{\max}$.

In order for nodes in V_i to decide the transmission powers for sending the IRP messages, we discuss the following two cases.

(1) For a node $j \in V_i$, if $P_j^{\max} \geq P_{ij}$, j can reach node i via the single-hop link $\overrightarrow{L_{ji}}$.

(2) If $P_j^{\max} < P_{ij}$, j must find a multi-hop path to reach i. There are at least three solutions. (a) j uses $P_j^{\max}$ to broadcast its IRP message with a special bit toggled to signal that the IRP may need to be relayed. When any other nodes receive such an IRP not addressed to themselves, they assist with relaying the message by re-broadcasting with their maximum transmission power. (b) j can send the IRP message via network layer packet routing protocols to i. Due to the previous assumption of a strongly connected $\overrightarrow{G}$, there exists a directed multi-hop path for node j to reach node i. A better approach in this case is to have j piggy-back the IRP message to i when it sends data packets to node i. (c) Node j takes advantage of the services provided by the sub-routing layer [8] to pass the IRP message back to i.

Having the knowledge of the locations and maximum transmission powers for itself and all its vicinity nodes, and under the assumption that the path loss models of all vicinity nodes are coherent, node i may derive the existence of the *vicinity edges*. For any two nodes $j, k \in V_i$, link $\overrightarrow{L_{jk}}$ is defined as one of i's vicinity edges, if $P_j^{\max} \geq P_{jk}$. Consequently, node i constructs its local *vicinity topology* that includes all its vicinity nodes, itself and the discovered *vicinity edges*. If node i's vicinity topology is denoted as $\overrightarrow{G_i}$, and the collection of its vicinity edges is denoted as $\overrightarrow{E_i}$, we obtain a weighted, directed graph with source vertex i and weight function $w : \overrightarrow{E_i} \rightarrow \mathbf{R}$:

$$\overrightarrow{G_i} = (V_i, \overrightarrow{E_i})$$

where the weight for each directed edge, $w(u_i, u_j)$, is the power required to reach u_j from u_i on the edge $u_i \rightarrow u_j$, equivalent to $P_{u_i u_j}$.

B. Phase 2: Deriving the minimum-power vicinity tree

With the knowledge of the weighted, directed topology $\overrightarrow{G_i}$, the weight, W_l, of a directed path $l = u_0 \rightarrow u_1 \rightarrow \ldots \rightarrow u_k$ from node u_0 to u_k is the sum of edge weights along the path, *i.e.*, $W_l = \sum_{m=1}^{k} w(u_{m-1}, u_m)$. The minimum power for node i to reach j is $\min(W_p)$ for all available paths p

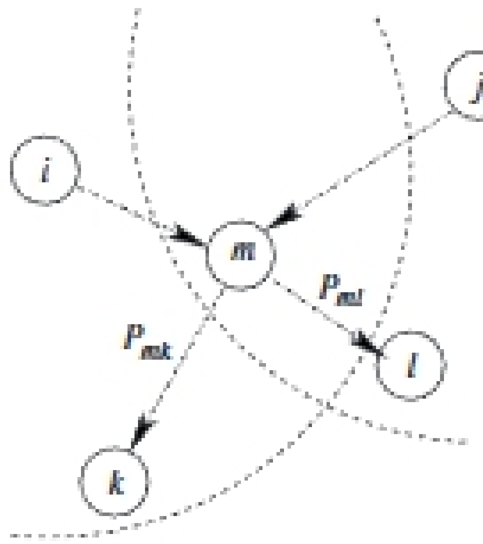

Fig. 1. An example of convergence on a node's transmission range

TABLE I

THE DISTRIBUTED TOPOLOGY CONTROL ALGORITHM

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To be executed at each node i ∈ V:
  Find G_i with full power P_i^max
  G_is = ShortestPath(G_i)
  for (each link E_jk ∈ G_is)
    Node i sends P_jk to node j

To be executed at each node j ∈ V_i:
  Extract P_jk from PR message from node i
  P_j = max(P_jk, P_j)
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from i to j. In essence, we attempt to find the shortest path in $\overrightarrow{G_i}$ from i to j. In this case, node i may execute a single-source shortest-paths algorithm, such as the Bellman-Ford or Dijkstra's algorithms (since edge weights are nonnegative), to derive the *minimum-power vicinity tree* $\overrightarrow{G_{is}} = (V_{is}, \overrightarrow{E_{is}})$. In fact, $\overrightarrow{G_{is}}$ is a typical shortest-paths tree from i to all other nodes in V_i, with the following additional properties:

- *Property 1.* Since there does not exist unreachable nodes in the in $\overrightarrow{G_i}$, we have $V_{is} = V_i$, and $\overrightarrow{E_{is}} \subseteq \overrightarrow{E_i}$.
- *Property 2.* The derivation of $\overrightarrow{G_{is}}$ depends solely on the edge weights, which does not assume a specific propagation model. However, with different path loss models, $\overrightarrow{G_{is}}$ may be different.

C. Phase 3: Propagation of transmission powers

In this phase, node i needs to calculate the transmission power needed for itself and each vicinity node in V_i, to ensure that all its minimum-power paths exist in the final minimum-power network topology. Specifically, for node i itself and each node in set V_i, the transmission power is assigned as the power required to reach the furthest one-hop downstream nodes in node i's minimum-power vicinity tree $\overrightarrow{G_{is}}$. Node i first adopts the minimum power assigned to itself, and then sends the minimum power required for each vicinity node with a explicit *power request* (PR) *message*.

Upon receiving the PR message, a vicinity node j compares the power requirement from i with its current power setting. If i requires a stronger transmission power at node j, node j increases its power accordingly. Otherwise, it discards the PR message. Note that its existing setting is assigned by itself or any other nodes that have executed the algorithm earlier than node i and propagated the PR message.

For example, in Fig. 1, we observe that node m is a vicinity node of both i and j, *i.e.*, $m \in \overrightarrow{V_{is}}$ and $m \in \overrightarrow{V_{js}}$. Given that $P_{mk} > P_{ml}$, if node i runs the algorithm first, P_m is set as P_{mk} by node i. When node j executes the algorithm at a later time, P_m is required to be P_{ml}. Although the current power setting on node m is $P_{mk} > P_{ml}$, node m can not reduce its transmission power to P_{ml} since it should not violate the reachability requirement in node i's minimum-power tree. Finally, our proposed algorithm is summarized in Table I.

D. Further Optimizations

The resulting minimum-power topology can be enhanced through the application of expiration or discard schemes of the PR message without violating the connectivity requirement. Particularly, (a) all PR messages expire at a node upon its completion of our algorithm; (b) a node discards the PR message that asks it to reach a node already listed as one of its vicinity nodes.

In the previous discussion, a node, such as node j, receives the PR messages from other nodes for it to act as routers for them in order to reach the downstream nodes in their minimum-power trees. However, we further observe that (a) that specific downstream node is one of node j's vicinity nodes; (b) node j may be able to find a shorter route to reach that downstream node through relaying at other vicinity nodes. Based on observation (a), as long as node j can reach that downstream node through a single-hop or multi-hop path, the connectivity for other nodes is not affected; based on the observation (b), in case node j finds a shorter path to that downstream node compared with the direct wireless link, it may further reduce its transmission power. Therefore, node j can safely discard all the PR messages it has received upon the completion of shortest path algorithm on its vicinity topology without violating the connectivity requirement from other nodes.

An example of the scenario is illustrated in Fig. 2. In this scenario, node A finds that the most power-efficient path to reach node C is $\overrightarrow{L_{AB}} \rightarrow \overrightarrow{L_{BC}}$ due to the fact that $P_{AB} + P_{BC} < P_{AC}$. As a result, node A sends a PR message to node B to have $P_B \geq P_{BC}$ such that node B can relay traffic from node A to node C. However, when node B executes the shortest path algorithm over its vicinity topology, it finds that rather than reaching node C directly, it is more power-efficient to reach node C via node D. This shorter path is unknown to node A as node D is not node A's vicinity node. Hence, node B sets its transmission power as P_{BD} rather than P_{BC} as indicated in the PR message from node A to achieve higher power efficiency. Eventually, node A is connected to node C through $\overrightarrow{L_{AB}} \rightarrow \overrightarrow{L_{BD}} \rightarrow \overrightarrow{L_{DC}}$.

Such expiration of PR messages does not imply that PR messages can be removed from our algorithm completely.