

Written Homework Assignment # 1

Due in class Feb. 12, 2002

These exercises are drawn from the following papers (all obtainable from <http://pantheon.yale.edu/~sz38/>) and from the related lectures and discussions in class:

- The Nisan-Ronen and Parkes papers in the Introduction section.
- All papers in the Routing section except Koutsoupias-Papadimitriou.

Problem 1 (15 points)

Recall that the algorithm given in the Feigenbaum-Papadimitriou-Sami-Shenker paper (FPSS) requires $O(d + d')$ stages to converge, where d' is as defined on page 9 of the paper, and d is the number of stages required for BGP to convergence. Give an infinite family of "bad examples" for this result. That is, show by giving an infinite family of graphs in which this is the case that d and d' can be, respectively, very small and very large; give an upper bound for d and a lower bound for d' (in terms of the number of nodes and edges) in this family of graphs.

Problem 2 (15 points)

Recall that the routing mechanism in the FPSS paper exhibits "overcharging" (a general problem with VCG routing mechanisms, as pointed out in Archer-Tardos). Quantify how bad the overcharging can be, and give a family of worst-case examples.

Problem 3 (15 points)

Recall that Roughgarden and Tardos claim that their formulation of "selfish routing" models IP and that min-cost routing models ATM. Why is the claim about IP an oversimplification? (Give as many reasons as you can.)

Problem 4 (5 points)

Why does the argument in Section 6 of the Hershberger-Suri paper (HS) not work for directed networks?

Problem 5 (15 points):

Recall that we observed in class that, after one pre-processing stage that can be accomplished within the required time bounds, each computation of w in Step 3(a) of the Path Algorithm in HS can be done in constant time in the case of path graphs. In the case of undirected networks, what is the analogous pre-processing step and how long does it take? In the case of directed networks, what is the analogous formula for w in Step 3(a), how should the network be pre-processed to enable constant-time computation of w , and how long does this pre-processing take?

Problem 6 (10 points)

Give a strategyproof mechanism for computing a Minimum Spanning Tree (MST) of a graph. The agents are the edges of the graph, and an agent's private type is the cost of the edge.

**** Problem 7 (25 points)**

Discuss the computational complexity, both centralized and distributed, of the MST mechanism you gave in Problem 6. That is, what is the best way of computing both the output (i.e., the MST) and the payments, in both a centralized computational model and a distributed computational model?